NEW AND ENHANCED CAPABILITIES IN RELEASE 8 OF THE SCA STATISTICAL SYSTEM

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CHAPTER 1

INTRODUCTION

This document serves as a supplement to the primary SCA user guides which document the well established capabilities in Release 7.0 of the SCA System and earlier. New and enhanced capabilities in Release 8.0 of the SCA System are described in this document along with the newer capabilities introduced in Release 7.3.

Release 8.0 of the SCA System is organized into editions designed to meet the needs of users with varying capability requirements and expertise in time series analysis and forecasting applications. The current SCA editions are

- Educational Edition
- Practitioner Edition
- Professional Edition (A)
- Professional Edition (B)
- Advanced Edition

For academic users teaching or learning the fundamentals of time series methods, the Educational Edition encapsulates time-tested capabilities in time series analysis and forecasting. Users who require automatic time series modeling and automatic outlier handling methods in addition to traditional user-directed methods will find the Practitioner Edition suited to their applications. This edition also appeals to users with large-scale forecasting applications or users interested in time series data mining and exploration.

For those users with interest in expanded capabilities, Professional Edition A or B of the SCA System add vector ARIMA and simultaneous transfer function models, causality tests, time-varying parameter models, segmented time series modeling and forecasting, GARCH and other nonlinear modeling, and more.

In an appendix to this document, all SCA System commands are summarized. The appendix indicates which SCA edition(s) the capability can be found, and the SCA user guide that best documents the capability's usage. A listing of the SCA System user guides is provided below:

(A) SCA Reference Manual for Fundamental Capabilities
(B) SCA Reference Manual for General Statistical Analysis
(C) Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 1
(D) Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 2
(E) GARCH Modeling using SCAB34S-GARCH and SCA WorkBench
(F) New and Enhanced Capabilities in Release 8 of the SCA Statistical System (*this document*)
(G) On-line Syntax Help (*Accessible through SCA WorkBench*)

An in-depth discussion of the forecasting and time series analysis capabilities in the SCA System are also provided in the book, *Time Series Analysis and Forecasting (Second Edition)* by Liu (2006). For those interested in using the SCA System in a classroom environment, we highly recommend this book. The book can be purchased on the SCA website (www.scausa.com).

New Features (Release 8.0)

The SCA System provides many new capabilities and enhancements related to the following topics:

- Time series power transformation analysis and diagnostics
- Improved forecasting using power transformations
- Time-varying parameter models
- Generalized threshold AR and ARIMA modeling
- Segmented time series modeling and forecasting
- GARCH modeling and application environment
- New seasonal ARIMA identification method
- Unit root testing
- Causality tests using vector ARIMA models
- Improved estimation with root checking of ARMA factors
- Date building, handling, indexing, and aggregation

These new capabilities of the SCA System are contained within select editions based on topic. We begin by summarizing the focus of the SCA System editions below.

Educational Edition

The SCA Educational Edition includes essential time series analysis and forecasting capabilities for teaching and learning. It is this fundamental module on which other SCA forecasting and time series products are built. The Educational Edition focuses on time-tested modeling capabilities, providing all the necessary tools to identify, estimate, diagnostically check, and forecast using various time series models. The SCA Educational Edition includes

- Box-Jenkins nonseasonal/seasonal ARIMA modeling
- New identification method for seasonal ARIMA models
- Powerful transfer function modeling and forecasting
- Effective LTF model identification for transfer functions
- Lagged regression with autocorrelated errors
- Intervention (impact) analysis
- Exponential smoothing using various methods
- Time series simulation
- Constrained parameter estimation

- A wide array of capabilities for general statistical analysis
- Large workspace (60,000 words) allocation

Practitioner Edition

The SCA Practitioner Edition builds on the Educational Edition by adding expert-system automatic time series modeling and forecasting features. In addition, it includes power packed capabilities to automatically detect and adjust for outliers during estimation which is a great tool for time series data mining. The Practitioner Edition offers an effective solution to handle repetitive modeling and forecasting tasks on a large number of time series. It is a natural choice in driving large scale forecasting applications that require automation. The SCA Practitioner Edition builds on the Educational Edition by adding

- Automatic identification of seasonal and nonseasonal ARIMA models
- Automatic transfer function modeling and intervention analysis
- Automatic detection and adjustment for outliers using a joint estimation algorithm by C. Chen and L.-M. Liu
- Automatically handles level shifts, temporary changes, additive, and innovational outliers
- Improved estimates of intervention effects through joint estimation of model parameters and outlier effects
- Better forecasting results by special handling of outliers occurring at the end of a time series
- Time series data mining and exploration
- Improve forecasting using power transformation
- Model identification and estimation with missing data
- Trading day and moving holiday adjustment
- Date functions to facilitate daily, weekly, and monthly modeling and forecasting
- Unrestricted workspace allocation

Professional Edition (A)

The SCA Professional Edition (A) builds on the Practitioner Edition by adding multivariate time series analysis and forecasting using vector ARIMA and simultaneous transfer function (STF) models. These advanced modeling approaches are well-suited to business, economic, industrial and social science time series data. The SCA Professional Edition (A) builds on the Practitioner Edition by adding

- Analysis and forecasting of multivariate time series using vector ARIMA models
- Causality tests using vector ARIMA models
- Analysis and forecasting of multivariate time series using simultaneous transfer function models that accommodate for intervention, trading day and moving holiday effects
- Analysis of spatial time series data
- Multivariate time series simulation using vector ARIMA and STF models

- Study of contemporaneous effects using structural form models employing STF models, or use reduced form vector ARMA models to leverage lagged dependencies
- Extend upon conventional econometric models by addressing serially correlated errors
- Model-based seasonal adjustment

Professional Edition (B)

The SCA Professional Edition (B) builds on the Practitioner Edition by adding power transformation, segmented time series modeling, nonlinear time series testing, identification, modeling, forecasting using TAR, threshold ARIMA, and threshold transfer function models. Also included are new analysis capabilities for time-varying parameter models and GARCH models. The SCA Professional Edition (B) adds capabilities for

- New criterion for power transformation to improve forecasting accuracy
- Segmented time series modeling and forecasting using weighted estimation methods
- Effective handling of clustered outliers, and desensitizing parameter estimates from temporary structural changes
- Threshold autoregressive (TAR) and general threshold ARIMA modeling and forecasting
- Piecewise and threshold transfer function modeling and forecasting
- Nonlinearity tests on time series
- Analysis using time-varying parameter models
- GARCH modeling (See SCAB34S GARCH below)

Advanced Edition

The Advanced Edition provides SCA’s full breadth of capabilities to model and forecast time series data combining the features of Professional Editions (A) and (B).

CHAPTER 2

DATE HANDLING AND TIME SERIES AGGREGATION

The SCA Statistical System (Practitioner Edition and above) includes capabilities for date handling and aggregation of time series. In this chapter, the commands DATEBUILD, DOWEEK, DAGGREGATE, and DVECTOR are discussed. These capabilities are particularly helpful for modeling and forecasting daily and weekly time series. Through these commands, time series can also be modeled in a more deterministic manner which often is useful in understanding the characteristics of a time series. Related capabilities including AGGREGATE, DAYS, DMATRIX, and EASTER are documented in the SCA reference manual, *Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 1*.

A date variable must be stored as a double precision variable in the SCA System. The DATEBUILD command automatically creates a date variable in double precision. If a date variable is read into the SCA System workspace from a file, the user must specify its precision as DOUBLE in the INPUT command. Storing a date variable with single precision will cause the date values to lose accuracy after the seventh digit. Dates are represented as 8-digit numbers of the format YYYYMMDD in the SCA System, where YYYY is the 4-digit year, MM is the 2-digit month, and DD is the 2-digit day. For example, August 1, 2007 is represented as the double precision number 20070801 in the SCA System.

2.1 Creating a Date Variable Using the DATEBUILD Command

The DATEBUILD command is used to generate a sequence of dates within a specified beginning period and ending period. An example of the DATEBUILD command follows. We begin by creating a yearly date variable beginning from 1970 to 2005. The following DATEBUILD command is specified

➔ DATEBUILD VYRS. BEGIN 1970. END 2005.

Alternatively, it is sometimes easier to specify the beginning period and the number of date values needed. This can be achieved using the command (the generated information is printed below)

➔ DATEBUILD VYRS. BEGIN 1970. NOBS 36.
➔ PRINT VYRS

```
VYRS    IS  A    36  BY     1  VARIABLE; DOUBLE PRECISION
1970.000  1971.000  1972.000  1973.000  1974.000  1975.000  1976.000
1977.000  1978.000  1979.000  1980.000  1981.000  1982.000  1983.000
1984.000  1985.000  1986.000  1987.000  1988.000  1989.000  1990.000
1991.000  1992.000  1993.000  1994.000  1995.000  1996.000  1997.000
1998.000  1999.000  2000.000  2001.000  2002.000  2003.000  2004.000
2005.000
```

We now illustrate the creation of quarterly, monthly, and daily dates using the DATEBUILD command. The BEGIN subcommand requires two arguments for quarterly and monthly dates. The first argument specifies the 4-digit year. The second argument specifies the beginning quarter or month. In the case of daily dates, a third argument is required to specify the beginning day.

➔ DATEBUILD VQTRS. BEGIN 1970,1. QUARTERLY. NOBS 36.
➔ DATEBUILD VMNTHS. BEGIN 1970,1. NOBS 36.
➔ DATEBUILD VDATES. BEGIN 1970,1,1. NOBS 36.
➔ PRINT VYRS, VQTRS, VMNTHS, VDATES. FORMAT '4F12.0'.

```
VYRS      IS  A    36  BY     1  VARIABLE; DOUBLE PRECISION
VQTRS     IS  A    36  BY     1  VARIABLE; DOUBLE PRECISION
VMNTHS    IS  A    36  BY     1  VARIABLE; DOUBLE PRECISION
VDATES    IS  A    36  BY     1  VARIABLE; DOUBLE PRECISION

VARIABLE        VYRS       VQTRS      VMNTHS       VDATES
  ROW
    1           1970      197001      197001    19700101
    2           1971      197002      197002    19700102
    3           1972      197003      197003    19700103
    4           1973      197004      197004    19700104
    5           1974      197101      197005    19700105
    6           1975      197102      197006    19700106
    7           1976      197103      197007    19700107
    8           1977      197104      197008    19700108
    9           1978      197201      197009    19700109
   10           1979      197202      197010    19700110
   11           1980      197203      197011    19700111
   12           1981      197204      197012    19700112
   13           1982      197301      197101    19700113
   14           1983      197302      197102    19700114
   15           1984      197303      197103    19700115
   16           1985      197304      197104    19700116
   17           1986      197401      197105    19700117
   18           1987      197402      197106    19700118
   19           1988      197403      197107    19700119
   20           1989      197404      197108    19700120
   21           1990      197501      197109    19700121
   22           1991      197502      197110    19700122
   23           1992      197503      197111    19700123
   24           1993      197504      197112    19700124
   25           1994      197601      197201    19700125
   26           1995      197602      197202    19700126
   27           1996      197603      197203    19700127
   28           1997      197604      197204    19700128
   29           1998      197701      197205    19700129
   30           1999      197702      197206    19700130
   31           2000      197703      197207    19700131
   32           2001      197704      197208    19700201
   33           2002      197801      197209    19700202
   34           2003      197802      197210    19700203
   35           2004      197803      197211    19700204
   36           2005      197804      197212    19700205
```

Weekly dates can be created from the daily dates using a two step procedure. The DATEBUILD command contains a HOLD subcommand that stores the components of the generated date variable. Through the HOLD subcommand, the user can store the year, quarter, month, day-of-month, and day-of-week information. Using the following commands, a weekly date variable (with the dates of Monday in each week) is created

➔ DATEBUILD VDATES. BEGIN 1970,1,1. NOBS 36. HOLD DOWEEK(VDOW).
➔ SELECT VDOW, VDATES. VALUES (1,1). NEW DOWT,WKBEGIN.

```
VARIABLE   VDOW   IS EDITED, THE RESULT IS STORED IN VARIABLE   DOWT
VARIABLE   DOWT   IS  A    5  BY   1  MATRIX
VARIABLE  VDATES  IS EDITED, THE RESULT IS STORED IN VARIABLE WKBEGIN
VARIABLE WKBEGIN  IS  A    5  BY   1  MATRIX
```

➔ PRINT WKBEGIN. FORMAT 'F12.0'.

```
  WKBEGIN  IS  A     5  BY     1  VARIABLE; DOUBLE PRECISION
    19700105
    19700112
    19700119
    19700126
    19700202
```

2.2 Extracting Day-of-Week Information from a Date Variable Using DOWEEK

In the previous example, we used the HOLD subcommand in DATEBUILD to store the day-of-week information of the generated date variable. Sometimes a double precision date variable (say VDATES) in the form YYYYMMDD already exists in the SCA workspace (e.g., read into the SCA workspace through the INPUT command). In such a case, the following DOWEEK command can be used to generate the day-of-week information

➔ DOWEEK VDATES. DAYOFWEEK VDOW.
➔ PRINT VDATES, VDOW. FORMAT 'F12.0, F4.0'.

```
 VDATES   IS  A    36  BY     1  VARIABLE; DOUBLE PRECISION
 VDOW     IS  A    36  BY     1  VARIABLE

VARIABLE       VDATES  VDOW
   ROW
     1        19700101   4
     2        19700102   5
     3        19700103   6
     4        19700104   7
     5        19700105   1
     6        19700106   2
     7        19700107   3
     8        19700108   4
     9        19700109   5
    10        19700110   6
    11        19700111   7
    12        19700112   1
```

```
     13        19700113   2
     14        19700114   3
     15        19700115   4
     16        19700116   5
     17        19700117   6
     18        19700118   7
     19        19700119   1
     20        19700120   2
     21        19700121   3
     22        19700122   4
     23        19700123   5
     24        19700124   6
     25        19700125   7
     26        19700126   1
     27        19700127   2
     28        19700128   3
     29        19700129   4
     30        19700130   5
     31        19700131   6
     32        19700201   7
     33        19700202   1
     34        19700203   2
     35        19700204   3
     36        19700205   4
```

The DOWEEK command can also extract the year, month, and day components of a date variable by specifying the optional DATEPARTS subcommand as illustrated below

```
➔ DOWEEK VDATES.  DAYOFWEEK VDOW. @
       DATEPARTS YRDATE, MNDATE, DYDATE.
➔ PRINT VDATES, VDOW, YRDATE, MNDATE, DYDATE.  FORMAT 'F12.0,4F7.0'
```

```
VARIABLE       VDATES   VDOW  YRDATE MNDATE DYDATE
   ROW
      1        19700101      4   1970      1      1
      2        19700102      5   1970      1      2
      3        19700103      6   1970      1      3
      4        19700104      7   1970      1      4
      5        19700105      1   1970      1      5
      6        19700106      2   1970      1      6
      7        19700107      3   1970      1      7
      8        19700108      4   1970      1      8
      9        19700109      5   1970      1      9
     10        19700110      6   1970      1     10
     11        19700111      7   1970      1     11
     12        19700112      1   1970      1     12
     13        19700113      2   1970      1     13
     14        19700114      3   1970      1     14
     15        19700115      4   1970      1     15
     16        19700116      5   1970      1     16
     17        19700117      6   1970      1     17
     18        19700118      7   1970      1     18
     19        19700119      1   1970      1     19
     20        19700120      2   1970      1     20
     21        19700121      3   1970      1     21
     22        19700122      4   1970      1     22
     23        19700123      5   1970      1     23
```

```
    24        19700124      6   1970      1     24
    25        19700125      7   1970      1     25
    26        19700126      1   1970      1     26
    27        19700127      2   1970      1     27
    28        19700128      3   1970      1     28
    29        19700129      4   1970      1     29
    30        19700130      5   1970      1     30
    31        19700131      6   1970      1     31
    32        19700201      7   1970      2      1
    33        19700202      1   1970      2      2
    34        19700203      2   1970      2      3
    35        19700204      3   1970      2      4
    36        19700205      4   1970      2      5
```

2.3 Building Dummy Variables Using the DVECTOR Command

The DVECTOR command is used to construct a set of dummy variables (design matrix) from the values of factor variables and stores the dummy variables as vector variables. The resultant dummy variables can then be used in subsequent time series modeling and forecasting.

The functionality of DVECTOR is similar to that of the DMATRIX command, but the resultant dummy variables are in vector form (as opposed to matrix form) and are more convenient to use in time series modeling and forecasting. By default, the dummy variables generated from DVECTOR are in a form such that their effect will be estimated as a deviation from the overall mean in a regression-like or time series model. The TYPE subcommand is used to specify whether a dummy variable will be created as a deviation from the model's mean (TYPE is DEVIATION); the first level contained in the factor variable is to be omitted (TYPE is T01); the last level contained in the factor variable is to be omitted (TYPE is T10); or whether a dummy variable should be generated for all levels in the factor variable (TYPE is T11). The default TYPE is DEVIATION.

To illustrate, the following DVECTOR command creates a set of dummy variables from the day-of-week factor variable VDOW. Using the default type (TYPE is DEVIATION), only six dummy variables are created since the last factor is confounded with the overall mean in a model. The root name for the dummy variables is defined in the MAIN-EFFECTS subcommand. Here, DY is specified as the root name which generates variable names DY1 to DY6 in our example.

```
➔ DVECTOR VDOW.  MAIN-EFFECTS DY.
➔ PRINT VDATES, VDOW, DY1 TO DY6.  FORMAT 'F12.0,8F4.0'
```

```
 VARIABLE       VDATES VDOW DY1 DY2 DY3 DY4 DY5 DY6
   ROW
     1        19700101   4   0   0   0   1   0   0
     2        19700102   5   0   0   0   0   1   0
     3        19700103   6   0   0   0   0   0   1
     4        19700104   7  -1  -1  -1  -1  -1  -1
     5        19700105   1   1   0   0   0   0   0
     6        19700106   2   0   1   0   0   0   0
     7        19700107   3   0   0   1   0   0   0
```

```
      8        19700108   4   0   0   0   1   0   0
      9        19700109   5   0   0   0   0   1   0
     10        19700110   6   0   0   0   0   0   1
     11        19700111   7  -1  -1  -1  -1  -1  -1
     12        19700112   1   1   0   0   0   0   0
     13        19700113   2   0   1   0   0   0   0
     14        19700114   3   0   0   1   0   0   0
     15        19700115   4   0   0   0   1   0   0
     16        19700116   5   0   0   0   0   1   0
     17        19700117   6   0   0   0   0   0   1
     18        19700118   7  -1  -1  -1  -1  -1  -1
     19        19700119   1   1   0   0   0   0   0
     20        19700120   2   0   1   0   0   0   0
     21        19700121   3   0   0   1   0   0   0
     22        19700122   4   0   0   0   1   0   0
     23        19700123   5   0   0   0   0   1   0
     24        19700124   6   0   0   0   0   0   1
     25        19700125   7  -1  -1  -1  -1  -1  -1
     26        19700126   1   1   0   0   0   0   0
     27        19700127   2   0   1   0   0   0   0
     28        19700128   3   0   0   1   0   0   0
     29        19700129   4   0   0   0   1   0   0
     30        19700130   5   0   0   0   0   1   0
     31        19700131   6   0   0   0   0   0   1
     32        19700201   7  -1  -1  -1  -1  -1  -1
     33        19700202   1   1   0   0   0   0   0
     34        19700203   2   0   1   0   0   0   0
     35        19700204   3   0   0   1   0   0   0
     36        19700205   4   0   0   0   1   0   0
```

In a later chapter, the DVECTOR command is used to generate a set of day-of-week dummy variables to explore time-varying parameters in time series models through the TVPEXPLORE command.

In some circumstances, the user may want to create the dummy variables in an alternative form. In such cases, the user can specify the keyword T11, T01, or T10 in the TYPE subcommand. In the next example, the T11 keyword is specified in the TYPE subcommand. This will create a dummy variable for each factor in the VDOW variable (DY1 to DY7).

➔ DVECTOR VDOW. MAIN-EFFECTS DY. TYPE T11.

```
THE MAIN-EFFECT DESIGN MATRIX FOR THE FACTOR   DOW     IS GENERATED.
 THE RESULT IS STORED IN VECTOR VARIABLES       DY# .  THE LEVELS ARE:
        1        2        3        4        5        6        7
```

➔ PRINT VDATES,VDOW, DY1 TO DY7. FORMAT 'F12.0,8F4.0'.

```
 VARIABLE       VDATES VDOW DY1 DY2 DY3 DY4 DY5 DY6 DY7
   ROW
     1        19700101   4   0   0   0   1   0   0   0
     2        19700102   5   0   0   0   0   1   0   0
     3        19700103   6   0   0   0   0   0   1   0
     4        19700104   7   0   0   0   0   0   0   1
     5        19700105   1   1   0   0   0   0   0   0
     6        19700106   2   0   1   0   0   0   0   0
     7        19700107   3   0   0   1   0   0   0   0
     8        19700108   4   0   0   0   1   0   0   0
     9        19700109   5   0   0   0   0   1   0   0
    10        19700110   6   0   0   0   0   0   1   0
    11        19700111   7   0   0   0   0   0   0   1
    12        19700112   1   1   0   0   0   0   0   0
    13        19700113   2   0   1   0   0   0   0   0
    14        19700114   3   0   0   1   0   0   0   0
    15        19700115   4   0   0   0   1   0   0   0
    16        19700116   5   0   0   0   0   1   0   0
    17        19700117   6   0   0   0   0   0   1   0
    18        19700118   7   0   0   0   0   0   0   1
    19        19700119   1   1   0   0   0   0   0   0
    20        19700120   2   0   1   0   0   0   0   0
    21        19700121   3   0   0   1   0   0   0   0
    22        19700122   4   0   0   0   1   0   0   0
    23        19700123   5   0   0   0   0   1   0   0
    24        19700124   6   0   0   0   0   0   1   0
    25        19700125   7   0   0   0   0   0   0   1
    26        19700126   1   1   0   0   0   0   0   0
    27        19700127   2   0   1   0   0   0   0   0
    28        19700128   3   0   0   1   0   0   0   0
    29        19700129   4   0   0   0   1   0   0   0
    30        19700130   5   0   0   0   0   1   0   0
    31        19700131   6   0   0   0   0   0   1   0
    32        19700201   7   0   0   0   0   0   0   1
    33        19700202   1   1   0   0   0   0   0   0
    34        19700203   2   0   1   0   0   0   0   0
    35        19700204   3   0   0   1   0   0   0   0
    36        19700205   4   0   0   0   1   0   0   0
```

2.4 Temporal Aggregation of a Time Series Using the DAGGREGATE Command

The DAGGREGATE command is used to generate a new time series through the temporal aggregation of a specified time series according to a companion date variable. The aggregated series will be organized based on a specified time interval such as year, quarter, month, week, or day. Aggregation method may be specified as the sum, mean, first, last, high or low of the data values in each date period.

In the following illustration of the DAGGREGATE command, we first generate values 1 to 36 and store them in a variable Y. The VDATES variable is used from the earlier example. The generated data is printed below.

```
➔ GENERATE Y.  NROWS 36.  PATTERN STEP(1,1).
➔ PRINT VDATES,Y.  FORMAT ‘F12.0, F7.1’.
```

```
VARIABLE       VDATES     Y
  ROW
     1        19700101    1.0
     2        19700102    2.0
     3        19700103    3.0
     4        19700104    4.0
     5        19700105    5.0
     6        19700106    6.0
     7        19700107    7.0
     8        19700108    8.0
     9        19700109    9.0
    10        19700110   10.0
    11        19700111   11.0
    12        19700112   12.0
    13        19700113   13.0
    14        19700114   14.0
    15        19700115   15.0
    16        19700116   16.0
    17        19700117   17.0
    18        19700118   18.0
    19        19700119   19.0
    20        19700120   20.0
    21        19700121   21.0
    22        19700122   22.0
    23        19700123   23.0
    24        19700124   24.0
    25        19700125   25.0
    26        19700126   26.0
    27        19700127   27.0
    28        19700128   28.0
    29        19700129   29.0
    30        19700130   30.0
    31        19700131   31.0
    32        19700201   32.0
    33        19700202   33.0
    34        19700203   34.0
    35        19700204   35.0
    36        19700205   36.0
```

Below, we use the DAGGREGATE command to aggregate the Y variable into a monthly time series YMNTHLY. The new monthly dates are stored in the variable MN and the number of cases aggregated in each period is stored in the variable MNNOBS.

```
➔ DAGGREGATE Y.  NEW YMNTHLY.  DATE VDATES.  METHOD MONTH(SUM). @
     HOLD DATE(MNDATE), NOBS(MNNOBS).
➔ PRINT MNDATE, YMNTHLY,MNNOBS.  FORMAT 'F12.0,F7.1,F7.0'
```

```
VARIABLE       MNDATE  YMNTHLY MNNOBS
   ROW
     1         197001  496.0     31
     2         197002  170.0      5
```

The DAGGREGATE command can also aggregate a series by year, quarter, week, and day. Next, we perform a weekly temporal aggregation of the Y series using the subcommand METHOD WEEK(LAST). The keyword LAST in parenthesis selects the last value in each period as the aggregation method.

When aggregating daily data into weekly periods, it is necessary to define the first day of the week. By default, DAGGREGATE uses Monday as the first day of the week. If a different first day of the week is required, the WBEGIN subcommand is employed to specify the first day of the week using 1=Monday, 2=Tuesday, …, or 7=Sunday. In this example, Monday is treated as the first day in the week if we set WBEGIN to 1 (the default value). If WBEGIN is set to 2, then Tuesday will be treated as the first day of the week, and so on.

In the following command, the weekly aggregated series is stored in YWEEK. The dates of the newly aggregated series are stored in WKDATE, and the number of cases aggregated within each period is stored in WKNOBS.

```
➔ DAGGREGATE Y.  NEW YWEEK. DATE VDATES.  METHOD WEEK(LAST). @
     WBEGIN 1.  HOLD DATES(WKDATE), NOBS(WKNOBS).
➔ PRINT WKDATE,YWEEK,WKNOBS.  FORMAT 'F12.0,F7.1, F7.0'.
```

```
VARIABLE       WKDATE  YWEEK  WKNOBS
   ROW
     1      1970010101    4.0      4
     2      1970010501   11.0      7
     3      1970011202   18.0      7
     4      1970011903   25.0      7
     5      1970012604   32.0      7
     6      1970020205   36.0      4
```

Notice that the WKDATE variable is a 10-digit number instead of an 8-digit number. When a series is aggregated into weekly periods, the 2-digit week-in-year value is appended to the usual date value in the format, YYYYMMDDWW. If the user wants to separate the week-in-year component from the regular date value, the following commands may be employed

```
➔ PROFILE PRECISION DOUBLE.
➔ WEEKDT=INT(WKDATE/100)
➔ WEEKNUM=MOD(WKDATE,100)
➔ PRINT WKDATE,WEEKDT,WEEKNUM.  FORMAT ‘2F12.0, F7.0’.
```

```
VARIABLE       WKDATE      WEEKDT  WEEKNUM
   ROW
     1      1970010101    19700101      1
     2      1970010501    19700105      1
     3      1970011202    19700112      2
     4      1970011903    19700119      3
     5      1970012604    19700126      4
     6      1970020205    19700202      5
```

The DAGGREGATE command also aggregates a series using the method of SUM, MEAN, FIRST, LAST, HIGH or LOW of the data values in each period being aggregated. The FIRST and LAST methods are well suited to aggregate financial time series where FIRST is the price on Monday and LAST is the price on Friday. The HIGH and LOW methods also have application in finance as well as other areas such as load forecasting where HIGH is considered as peak load.

CHAPTER 3

UNIT ROOT TESTS

The SCA System provides the UROOT command to perform unit root tests on a time series following the works by Dickey and Fuller (1979,1981), Phillips and Perron (1988), and others.

3.1 Dickey-Fuller Unit Root Tests

To begin discussion of unit root testing in the SCA System, first consider the following regression model that represents a first-order autoregressive AR(1) process:

$$Y_t = \rho Y_{t-1} + a_t \tag{3.1}$$

Subtracting Y_{t-1} from both sides of equation (3.1), the above regression model can be recast into its equivalent form

$$\nabla Y_t = \gamma Y_{t-1} + a_t \tag{3.2}$$

where $\gamma = \rho - 1$.

The regression model in (3.2) can be estimated using ordinary least squares (OLS) and the one-tailed unit root test is based on the estimated parameter where the null and alternative hypotheses are

$$\begin{array}{ll} H_0 : \gamma = 0 & (Y_t \text{ contains a unit root}) \\ H_1 : \gamma < 0 & (Y_t \text{ is stationary}). \end{array}$$

In addition to the unit root regression model in (3.2), Dickey and Fuller (1979) consider two alternative regression models for time series that can be represented by an AR(1) process. The regression models are

$$\nabla Y_t = \mu + \gamma Y_{t-1} + a_t \tag{3.3}$$

$$\nabla Y_t = \mu + \beta t + \gamma Y_{t-1} + a_t \tag{3.4}$$

Whereas the regression model in (3.2) is appropriate to test a time series that follows a pure random walk process, the regression model in (3.3) adds a constant parameter μ to better represent a time series that exhibits drift. Further, if a time series exhibits drift and linear trend, the regression model in (3.4) extends (3.3) by including the time trend parameter β.

In the SCA System, the regression model in (3.2) is referred to as the simple Dickey-Fuller test (DF), the regression model in (3.3) is referred to as the Augmented Dickey-Fuller test with constant (DFC), and the regression model in (3.4) is referred to as the Augmented Dickey-Fuller test with both constant and trend (DFT).

If a time series cannot be adequately represented by an AR(1) process, the regression models in (3.3) and (3.4) can be expanded to include higher-order autoregressive terms, AR(p), as shown below:

$$\nabla Y_t = \mu + \gamma Y_{t-1} + \phi_1 \nabla Y_{t-1} + \phi_2 \nabla Y_{t-2} + \ldots + \phi_p \nabla Y_{t-p} + a_t \tag{3.5}$$

$$\nabla Y_t = \mu + \beta t + \gamma Y_{t-1} + \phi_1 \nabla Y_{t-1} + \phi_2 \nabla Y_{t-2} + \ldots + \phi_p \nabla Y_{t-p} + a_t \tag{3.6}$$

The common parameter γ in the above regression models is the key to the Dickey-Fuller unit root tests. After the model is estimated using ordinary least squares (OLS) method, if γ is not statistically different from zero, the time series is determined to contain a unit root. The test statistic for γ is the t-statistic derived from the OLS estimation, where

$$\text{t-statistic} = \hat{\gamma}/s_{\gamma}\ . \tag{3.7}$$

However, the critical values to test the hypothesis of a unit root do not follow a typical t-distribution. Dickey and Fuller (1981) derive a non-standard set of critical values to account for bias in the OLS estimators. The SCA UROOT command uses the critical values that are interpolated from the Dickey-Fuller tables to test the hypothesis of a unit root.

It is important to note that the asymptotic distribution of the t-statistic on $\hat{\gamma}$ is independent of the number of lagged first-order differences (∇Y_{t-i}'s) included in the Augmented Dickey-Fuller testing models in (3.5) and (3.6). Therefore, the nonstandard critical value tables derived by Dickey-Fuller are still applicable when lagged first-order difference terms are included in the Augmented Dickey-Fuller models.

The usual *i.i.d.* assumptions apply on the Dickey-Fuller unit tests. However, these assumptions are relaxed under the Phillips-Perron method of unit root testing.

Mills (1993) indicates that it is desirable to include enough lagged first-order differences in the Dickey-Fuller regression models because as the sample size increases, the effects of the correlation structure of the residuals become more precise on the shape of the distribution of the non-standard critical values. The exact number of lagged first-order differences included in the regression model is arbitrary, and may be influenced by both sample size and seasonality of the time series. For example, Schwert (1987) recommends setting the number of lags (p) to

$$\text{INT}\left(s\left(n/100\right)^{0.25}\right),$$

where “s” is the seasonal lag (e.g., 12 for monthly seasonality) and “n” is the sample size. On the other hand, Diebold and Nerlove (1990) recommend that in practice the simpler function

$$\text{INT}\left(n^{0.25}\right),$$

is adequate and works well in practice.

3.2 Phillips-Perron Unit Root Tests

Phillips and Perron (1988) presented an alternative unit root test method based on nonparametric methods. By doing so, the *i.i.d.* assumptions on the innovations of the estimated regression models can be relaxed. This is appealing to those working with financial time series, since it is often desirable to allow for heterogeneity of the variance. The Phillips-Perron unit root test is based on the same models employed by Dickey-Fuller, namely the equations in (3.2) - (3.4), to obtain the estimate of γ. Since lagged differencing terms are not included in these regression models, Phillips-Perron provide a correction for general forms of serial correlation and heterogeneity in the computation of the test statistic for γ .

The derivation of the Phillips-Perron unit root test will not be detailed in this document. For those interested in its derivation, the original works of Phillips (1987) and Phillips and Perron (1988) are recommended. Hamilton (1994) also provides a detailed discussion regarding this topic.

To provide a brief overview of the Phillips-Perron unit root test, consider the regression equation in (3.3), which is repeated below for convenience:

$$\nabla Y_t = \mu + \gamma Y_{t-1} + a_t$$

An OLS estimation is performed on this model to obtain an estimate of γ without regard to possible serial correlation in the residuals, $\hat{a}_t$. Given γ, Phillips proposes that the test statistic be adjusted to correct for serial correlation and/or the lack of constant variance in the residuals as shown below

$$z\left(\tau_\mu\right) = \tau_\mu\left(\hat{\sigma}/\hat{\sigma}_{\tau j}\right) - 0.5\left(\hat{\sigma}_{\tau j}^2 - \hat{\sigma}^2\right) n \left\{\hat{\sigma}_{\tau j}^2 \sum_{t=2}^{n}\left(Y_{t-1} - \bar{Y}_{-1}\right)^2\right\}^{-0.5} \tag{3.8}$$

where $\hat{\sigma}^2$ is the sample variance of a_t

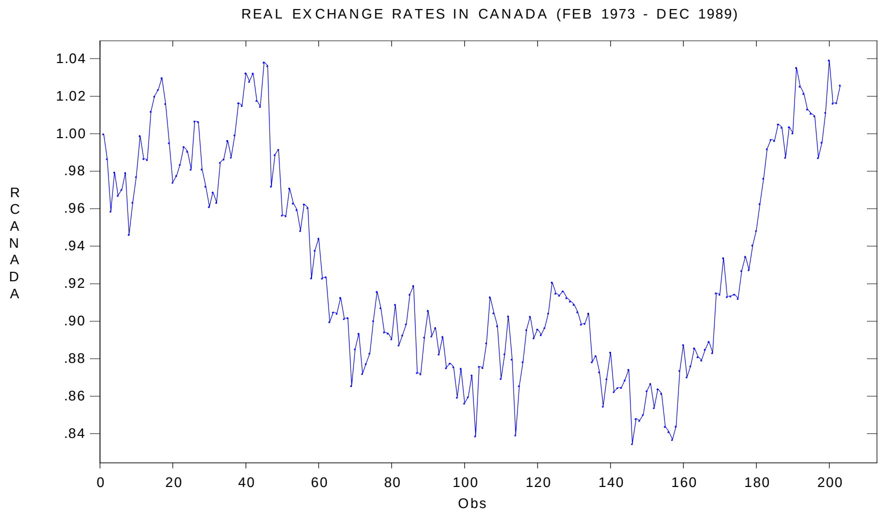
$$\bar{Y}_{-1} = (n-1)^{-1} \sum_{t=1}^{n-1} Y_t, \quad \text{and}$$

$$\hat{\sigma}_{\tau j}^2 = n^{-1} \sum_{t=1}^{n} a_t^2 + 2n^{-1} \sum_{i=1}^{j} \omega_{ij} \sum_{t=i+1}^{n} a_t a_{t-i}$$

3.3 Example: Canadian Real Exchange Rate

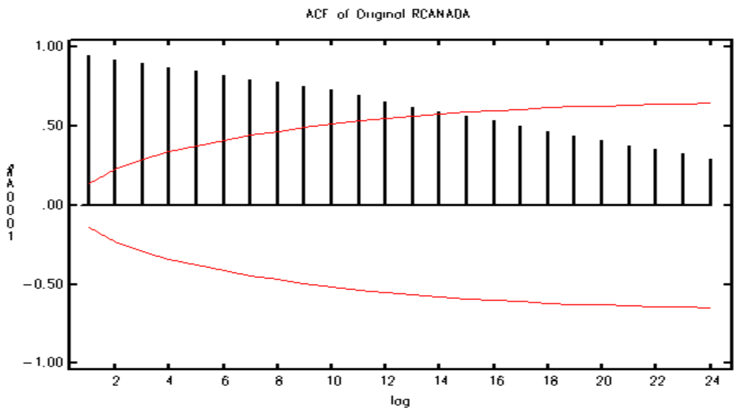
In this section we illustrate unit root tests using an example of Canadian real exchange rates (RCANADA). The data consist of monthly series from February 1973 up to and including December 1989 (a total of 203 observations), see Enders (1995, page 261) for more information regarding this time series.

Time Series Plot of Real Exchange Rates in Canada

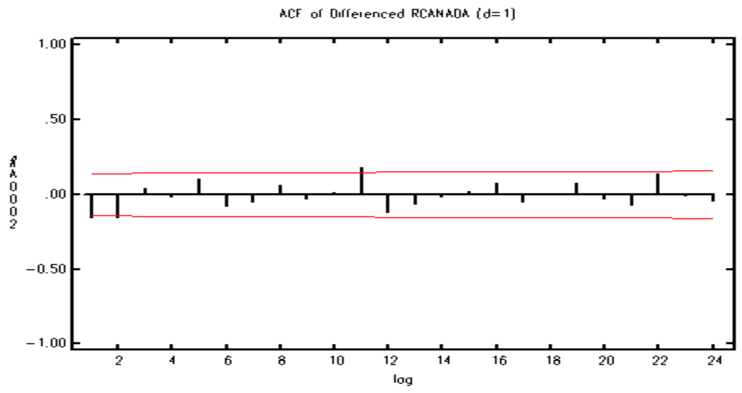


The sample autocorrelation functions of the original series and the first-order difference of the RCANADA series are:

Sample ACF of Real Exchange Rates in Canada (Original)



Sample ACF of Real Exchange Rates in Canada (1st Order Difference)



We are interested in testing whether a unit root is present using both the Augmented Dickey-Fuller and Phillips-Perron tests. After the data are brought into the SCA System, we begin by performing an Augmented Dickey-Fuller test with a constant term (DFC) on the RCANADA series. The equation used for this test is presented in (3.5). It is shown below for convenience

$$\nabla Y_t = \mu + \gamma Y_{t-1} + \phi_1 \nabla Y_{t-1} + \phi_2 \nabla Y_{t-2} + \ldots + \phi_p \nabla Y_{t-p} + a_t \, .$$

The UROOT command is specified using the DFC method and orders of 0 and 12. The order of 0 denotes that no lagged first-order differences will be employed in the above model which assumes that the time series follows a random walk process. The order of 12 denotes that lagged first-order differences of 1 through 12 will be employed to accommodate for higher-order autoregressive processes in the residual series.

```
➔  UROOT RCANADA.  METHOD IS DFC.  ORDERS ARE  0, 12.
```

```
AUGMENTED DICKEY-FULLER TEST WITH CONSTANT
SERIES TO BE TESTED FOR UNIT ROOT(S) IS:   RCANADA
TEST BASED ON SAMPLE SIZE:                     203
PROBABILITY OF A SMALLER VALUE IS SET TO:    0.050

                     (<----  INTERPOLATED D-F TABLE  ---->)
                    CRITICAL VALUE BASED ON OLS t-STATISTIC
           TEST       0.010     0.050     0.100     0.050
ORDER   STATISTIC     LEVEL     LEVEL     LEVEL     LEVEL   UROOT
    0       -1.81     -3.48     -2.88     -2.57     -2.88     YES
   12       -1.51     -3.48     -2.88     -2.57     -2.88     YES

                     (<----  INTERPOLATED D-F TABLE  ---->)
                      CRITICAL VALUE BASED ON OLS AR COEFF.
           TEST       0.010     0.050     0.100     0.050
ORDER   STATISTIC     LEVEL     LEVEL     LEVEL     LEVEL   UROOT
    0       -7.15    -20.14    -13.91    -11.14    -13.91     YES
   12       -7.23    -20.14    -13.91    -11.14    -13.91     YES
```

Upon executing the UROOT command, the above results are obtained. The output provides two forms of the DFC test statistic and corresponding critical values across multiple significance levels. If a significance level is not specified by the user, the UROOT command bases the one-tailed hypothesis test on a critical value of 0.05 (i.e., 5% one-tailed test) which are interpolated from the published tables of Dickey and Fuller (1981). The upper portion of the UROOT table, which is based on the OLS t-statistic, indicates that the null hypothesis of a unit root cannot be rejected at the 5% level with lag 0 (t=-1.81) or lags 1 to 12 (t=-1.51). Therefore, we conclude that a unit root is present in the RCANADA time series. The second table provides the unit root test results based on the estimated AR parameters of the Dickey-Fuller regression model.

It may be interesting to compare the results from the Augmented Dickey-Fuller test with constant (DFC) with the results of the corresponding Phillips-Perron test with constant (PPC). The same model used by the DFC method is also used for the PPC method. The difference is that Phillips and Perron (1988) adjust the test statistic using a nonparametric approach.

➔ UROOT RCANADA. METHOD IS PPC. ORDERS ARE 0, 12.

```
AUGMENTED PHILLIPS-PERONE TEST WITH CONSTANT
SERIES TO BE TESTED FOR UNIT ROOT(S) IS:   RCANADA
TEST BASED ON SAMPLE SIZE:                     203
PROBABILITY OF A SMALLER VALUE IS SET TO:    0.050

                     (<----  INTERPOLATED D-F TABLE  ---->)
                    CRITICAL VALUE BASED ON OLS t-STATISTIC
           TEST       0.010     0.050     0.100     0.050
ORDER   STATISTIC     LEVEL     LEVEL     LEVEL     LEVEL   UROOT
    0       -1.81     -3.48     -2.88     -2.57     -2.88     YES
   12       -1.59     -3.48     -2.88     -2.57     -2.88     YES

                     (<----  INTERPOLATED D-F TABLE  ---->)
                      CRITICAL VALUE BASED ON OLS AR COEFF.
           TEST       0.010     0.050     0.100     0.050
ORDER   STATISTIC     LEVEL     LEVEL     LEVEL     LEVEL   UROOT
    0       -7.15    -20.14    -13.91    -11.14    -13.91     YES
   12       -5.62    -20.14    -13.91    -11.14    -13.91     YES
```

Notice that the same critical values proposed by Dickey-Fuller are used for the Phillips-Perron test. However, the test statistic for lag 12 (t=-1.59) differs slightly from the test statistic obtained by using the DFC method (t=-1.51). Both methods conclude that the real exchange rate of Canada has a unit root at the 5% level.

CHAPTER 4

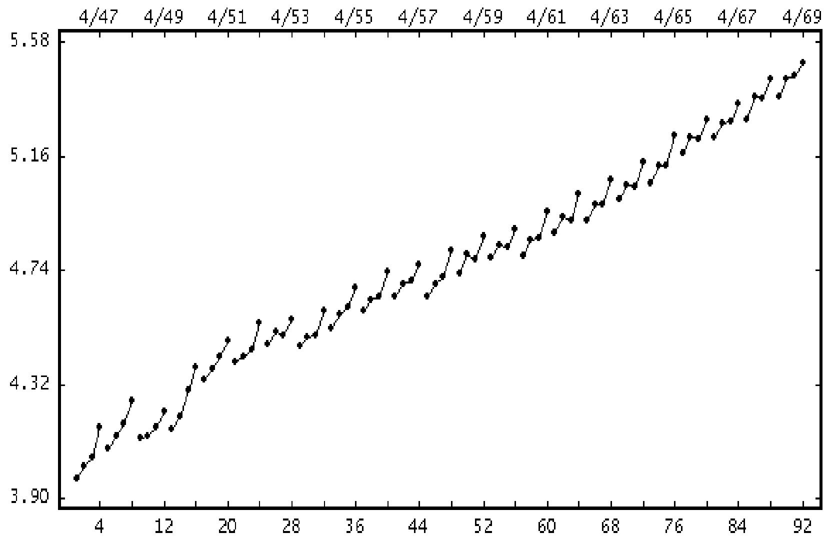
NEW IDENTIFICATION METHOD FOR SEASONAL TIME SERIES

The SCA Statistical System (Educational Edition and above) includes a new method of model identification for seasonal or periodic time series. In this chapter, the RSFILTER command is presented. A few supporting commands, ACF, PACF, TSMODEL, and ESTIM, are also used to complete the RSFILTER examples. Related capabilities to RSFILTER including IARIMA and IESTIM are documented in the SCA reference manual, *Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 2*.

The RSFILTER command is intended as an educational tool for seasonal time series model identification. RSFILTER implements the filtering identification method (Liu, 1989) to generate a pure nonseasonal (R_t) series and a pure seasonal (S_t) series from the original series. The models for the R_t and S_t series can then be easily identified, leading to simplification of seasonal model identification. The IARIMA command, discussed later in this document, internally uses the filtering identification method to automatically identify time series models. The RSFILTER is intentionally designed not to provide the user with the final model.

To illustrate the RSFILTER command, we begin by displaying a graph of the log transformed quarterly nominal gross national product (GNP, in billions of current dollars) of the United States between the first quarter of 1947 and the fourth quarter of 1969. This series will be referred to by LNGNP in this example. This series is not seasonally adjusted; therefore we observe distinct recurring patterns from year to year. A time series plot of the LNGNP series is displayed using SCAGRAF by issuing the command

```
➔ GRAPH LNGNP.  TYPE TSPLOT.
```



The printed values of the LNGNP series are displayed below for reference.

	First quarter	Second quarter	Third quarter	Fourth quarter
1947	3.9778	4.0236	4.0518	4.1667
1948	4.0843	4.1287	4.1790	4.2584
1949	4.1239	4.1304	4.1667	4.2195
1950	4.1589	4.2017	4.2973	4.3883
1951	4.3386	4.3808	4.4212	4.4853
1952	4.4067	4.4224	4.4509	4.5475
1953	4.4705	4.5142	4.5031	4.5602
1954	4.4601	4.4965	4.4998	4.5911
1955	4.5283	4.5788	4.6092	4.6784
1956	4.5911	4.6338	4.6454	4.7336
1957	4.6482	4.6923	4.7059	4.7622
1958	4.6434	4.6895	4.7158	4.8122
1959	4.7283	4.7991	4.7816	4.8668
1960	4.7916	4.8331	4.8251	4.8911
1961	4.7925	4.8536	4.8606	4.9572
1962	4.8775	4.9388	4.9280	5.0206
1963	4.9258	4.9843	4.9870	5.0764
1964	5.0006	5.0569	5.0518	5.1393
1965	5.0639	5.1305	5.1293	5.2402
1966	5.1716	5.2332	5.2274	5.2973
1967	5.2284	5.2842	5.2903	5.3552
1968	5.2978	5.3813	5.3744	5.4467
1969	5.3822	5.4485	5.4587	5.5074

Traditional means to identify an ARIMA model for time series typically involve interpretation of time series plots, autocorrelation functions (ACF), partial autocorrelation functions (PACF), and extended autocorrelation functions (EACF) of the time series. Whereas it is usually not difficult to identify models for nonseasonal time series using these tools, the task of model identification becomes more complex when seasonal time series are involved due to the *interaction patterns* of seasonal and nonseasonal information generated by these identification tools. By generating pure seasonal and nonseasonal series using RSFILTER, traditional model identification methods can be employed on these component series easily.

The RSFILTER command first determines if differencing (seasonal and nonseasonal) is required to induce stationarity. This step is determined automatically by RSFILTER based on the patterns of sample ACF's and parameter estimates of the intermediary filtering models. The filtering method uses an intermediary model of the form $\mathrm{ARMA}(1,1) \times \mathrm{ARMA}(1,1)_s$. Once the differencing orders are determined, the nonseasonal (R_t) and seasonal (S_t) component series are obtained by filtering the differenced Y_t series (denoted as y_t) such that

$$y_t = C_1 + \left(1-\Theta_1 B^s\right) \big/ \left(1-\Phi_1 B^s\right) R_t, \text{ and} \tag{4.1}$$

$$y_t = C_2 + \left(1-\theta_1 B\right) \big/ \left(1-\phi_1 B\right) S_t \tag{4.2}$$

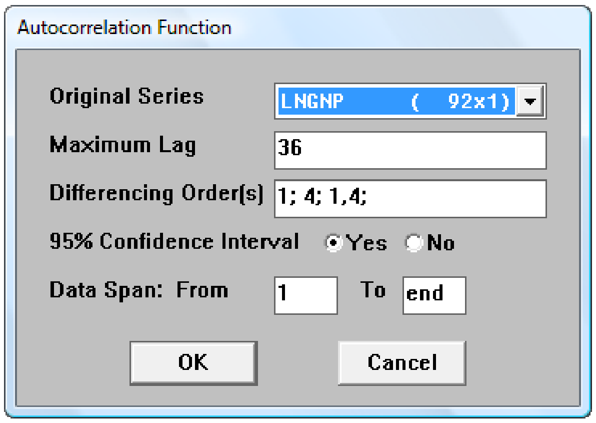
Note that R_t is like the residual series generated using (4.1) and S_t is like the residual series generated using (4.2).

Once the R_t and S_t series are generated as pure nonseasonal and pure seasonal component series using this filtering method, model identification becomes much easier. The RSFILTER command also displays the intermediary models which provide the user with information regarding the differencing orders selected by RSFILTER. The user can also employ alternative differencing in the RSFILTER command by specifying the DFORDER and NODFORDER subcommands.

We continue the example by displaying autocorrelation function (ACF) plots of the LNGNP series with no differencing, first-order differencing, fourth-order differencing, and first and fourth-order differencing using the command

```
➔ GRAPH LNGNP.  TYPE ACF. PAUSE
```

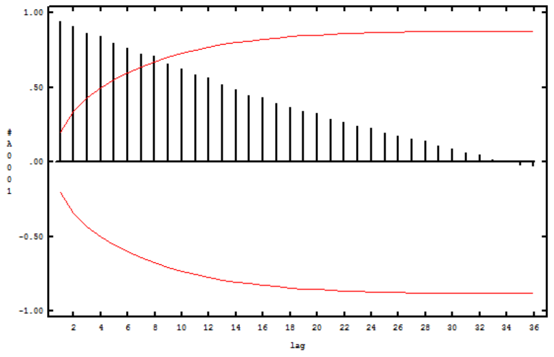
The PAUSE subcommand allows the user to overwrite the options of the ACF plots generated. The following dialog box will be displayed where the differencing orders scenarios are specified for a quarterly time series



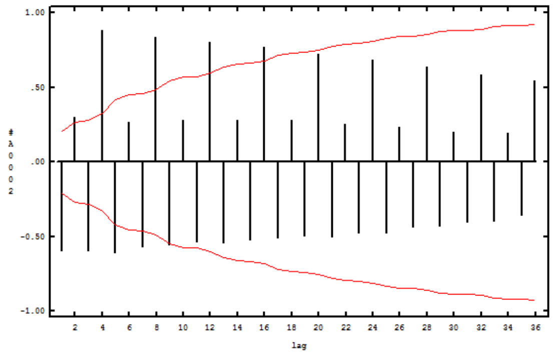
Original Series: LNGNP (92x1)
Maximum Lag: 36
Differencing Order(s): 1; 4; 1,4;
95% Confidence Interval: [✓ Yes] No
Data Span: From 1 To end

Clicking on the OK button then displays the ACF plots below that are typically used for ARIMA model identification.

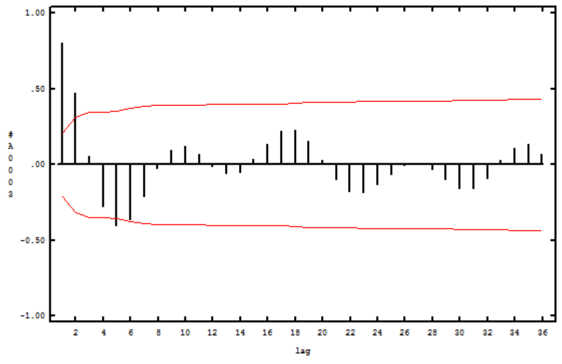
ACF (No Differencing)



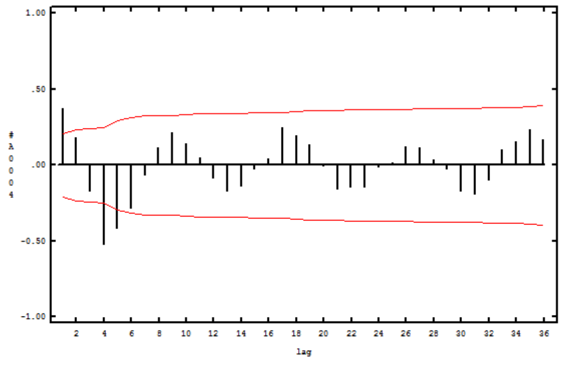
ACF (First-order Differencing)



ACF (Fourth-order Differencing)



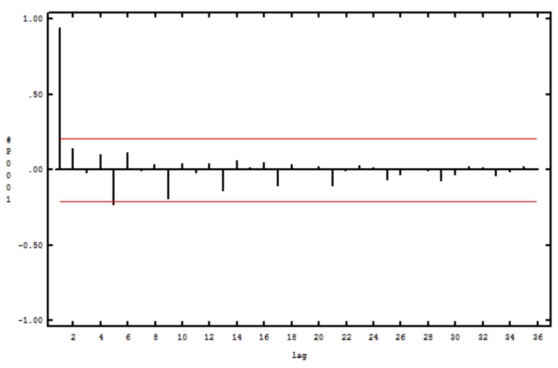
ACF (First and Fourth-order Differencing)



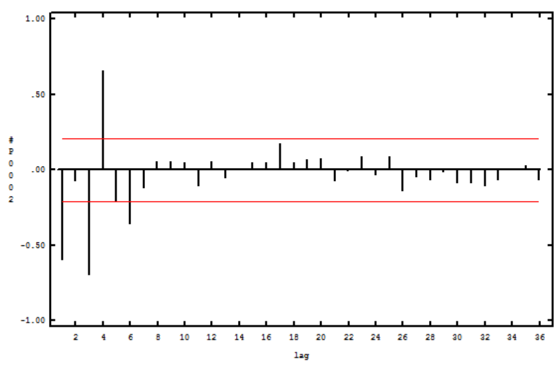
Partial Autocorrelation function (PACF) plots of the LNGNP series with no differencing, first-order differencing, fourth-order differencing, and first and fourth-order differencing are also displayed in SCAGRAF using the command

➔ GRAPH LNGNP. TYPE PACF. PAUSE

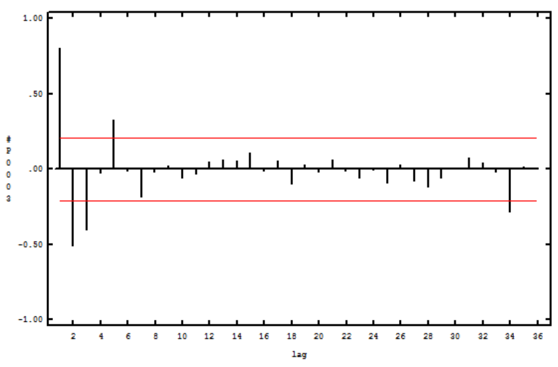
PACF (No Differencing



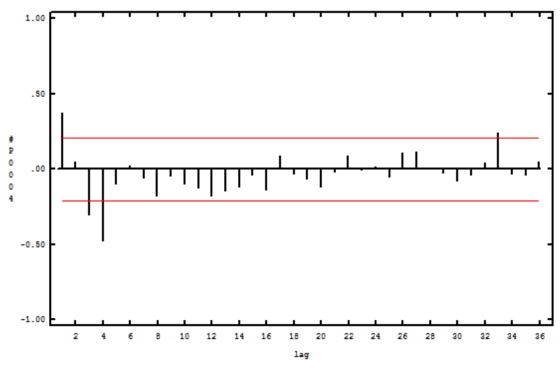
PACF (1st Order Differencing)



PACF (4th Order Differencing)



PACF (1st and 4th Order Differencing)



Using traditional identification methods, the ACF plots suggest a fourth-order differencing is appropriate for the LNGNP series. A first-order differencing is <u>not needed</u> since the ACF plot does not possess a slow die-out pattern after a fourth-order differencing is employed. We now explore the RSFILTER capability to help in ARIMA model identification.

The RSFILTER command shown below is employed to assist in the identification of an ARIMA model for the LNGNP series. Potential seasonality is specified using the SEASONALITY subcommand. In this example, the LNGNP series is a quarterly time series and therefore a potential seasonality of four is specified. Failure to specify potential seasonality in the RSFLITER command will result in the use of inappropriate intermediate models and produce undesirable results. The nonseasonal and seasonal component series are saved under the NEW-SERIES subcommand.

➔ RSFILTER LNGNP. NEW-SERIES R,S. SEASONALITY 4.

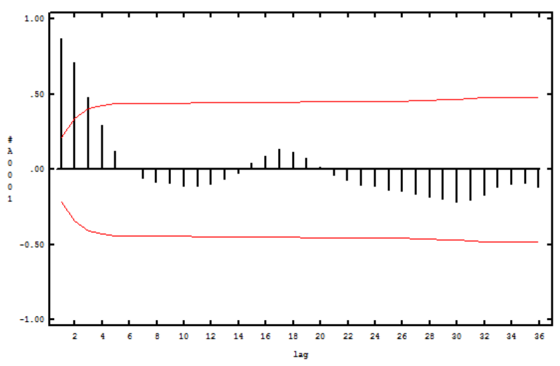
```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- UTSMODEL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

 LNGNP     RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1                    CNST      1      0     NONE    14.0528   14.4065    .98
  2           LNGNP     MA       1      1     NONE     -.1715     .1305  -1.31
  3           LNGNP     MA       2      4     NONE      .6008     .1033   5.81
  4           LNGNP    D-AR      1      1     NONE      .8402     .0732  11.47
  5           LNGNP    D-AR      2      4     NONE      .9936     .0115  86.75

TOTAL NUMBER OF OBSERVATIONS . . . .            92
EFFECTIVE NUMBER OF OBSERVATIONS . .            87
RESIDUAL STANDARD ERROR. . . . . . .  0.190659E-01

PARAMETER ESTIMATES DURING DIFFERENCING DETERMINATION
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- UTSMODEL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                       4
 LNGNP     RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1                    CNST      1      0     NONE      .0623     .0085   7.36
  2           LNGNP     MA       1      1     NONE     -.1312     .1433   -.92
  3           LNGNP     MA       2      4     NONE      .2811     .1872   1.50
  4           LNGNP    D-AR      1      1     NONE      .8531     .0843  10.13
  5           LNGNP    D-AR      2      4     NONE     -.3206     .1743  -1.84

TOTAL NUMBER OF OBSERVATIONS . . . .            92
EFFECTIVE NUMBER OF OBSERVATIONS . .            83
RESIDUAL STANDARD ERROR. . . . . . .  0.190460E-01
```

The output displayed by the RSFILTER command provides two model summaries. The initial model summary shows the filtering model prior to its determination of differencing required to induce stationarity. The second model summary shows the filtering model after differencing has been determined. Based on this second model, the R_t and S_t series are generated.

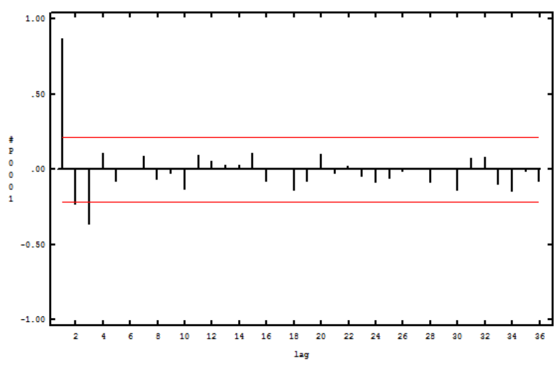
We now direct our attention to model identification of the nonseasonal R_t series. The IDEN keyword for the TYPE subcommand in the following GRAPH command will generate both ACF and PACF plots.

➔ GRAPH R. TYPE IDEN. PAUSE.

ACF (No Differencing)



PACF (No Differencing)



The extended autocorrelation function (EACF) is also a valuable tool in identifying ARIMA models. The EACF table is computed using the EACF command

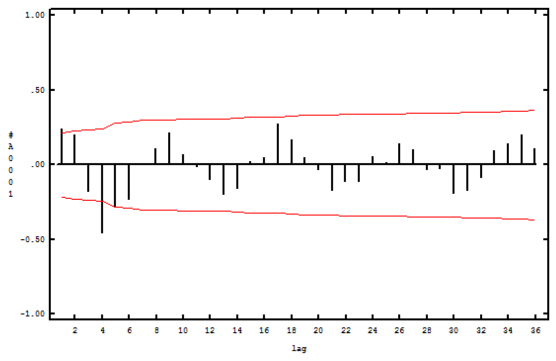
➔ EACF R.

```
THE EXTENDED ACF TABLE
(Q-->)   0    1    2    3    4    5    6    7    8    9   10   11   12
-------------------------------------------------------------------------
(P= 0)  .87  .71  .48  .30  .12  .01 -.05 -.08 -.09 -.11 -.11 -.10 -.06
(P= 1)  .24  .38  .18  .22  .13  .02 -.09 -.08 -.02 -.11 -.03 -.10 -.05
(P= 2) -.47  .35 -.20  .20 -.05 -.10  .07 -.08  .02 -.12  .04 -.11  .06
(P= 3)  .17 -.15  .00  .19 -.06 -.12 -.04 -.03  .17  .05  .02  .05  .00
(P= 4)  .45 -.20  .01  .04 -.07 -.04  .02 -.03  .15 -.03  .01  .04 -.00
(P= 5)  .35  .26 -.10 -.06 -.01 -.05  .04 -.03  .14  .01 -.01  .04 -.01
(P= 6) -.30  .38 -.44  .03 -.03 -.00  .00 -.02  .10 -.03 -.08  .06 -.02

SIMPLIFIED EXTENDED ACF TABLE (5% LEVEL)
(Q-->)  0  1  2  3  4  5  6  7  8  9 10 11 12
----------------------------------------------
(P= 0)  X  X  X  0  0  0  0  0  0  0  0  0  0
(P= 1)  X  X  0  0  0  0  0  0  0  0  0  0  0
(P= 2)  X  X  0  0  0  0  0  0  0  0  0  0  0
(P= 3)  0  0  0  0  0  0  0  0  0  0  0  0  0
(P= 4)  X  0  0  0  0  0  0  0  0  0  0  0  0
(P= 5)  X  0  0  0  0  0  0  0  0  0  0  0  0
(P= 6)  X  X  X  0  0  0  0  0  0  0  0  0  0
```

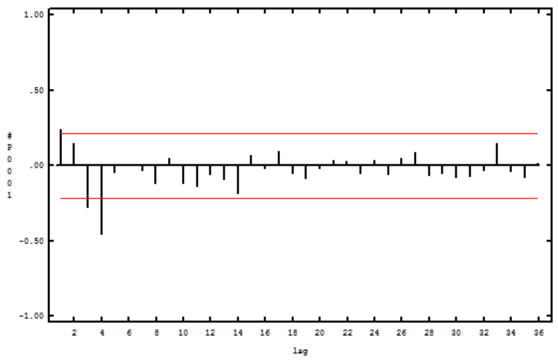
Based on the ACF, PACF, and EACF of the nonseasonal component series, an AR(3) or ARMA(1,2) model may be appropriate for R_t. We now produce the ACF and PACF plots for the seasonal component series, S_t

➔ GRAPH S. TYPE IDEN. PAUSE.

ACF (No Differencing)



PACF (No Differencing)



Based on the ACF and PACF of the seasonal component series, a seasonal MA(1)$_4$ model is appropriate for S_t. Now, the models for the nonseasonal and seasonal component series can be combined. We specify the first candidate model as an ARIMA(3,0,0) x ARIMA(0,1,1)$_4$ using the TSMODEL command,

```
➔ TSMODEL MODELA.  MODEL IS @
     LNGNP(4)=C1+(4 ; TH4A)/(1,2,3 ; PH1A,PH2A,PH3A)NOISE.
```

The model is then estimated using exact maximum likelihood using the ESTIMATE command shown below.

```
➔ ESTIMATE MODELA.  METHOD EXACT.  HOLD RESIDUALS(RESA).

SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODELA
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        4
 LNGNP     RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
  1    C1              CNST      1       0     NONE      .0609     .0041  14.91
  2   TH4A    LNGNP     MA       1       4     NONE      .4237     .1123   3.77
  3   PH1A    LNGNP    D-AR      1       1     NONE     1.0437     .1005  10.38
  4   PH2A    LNGNP    D-AR      1       2     NONE      .0181     .1516    .12
  5   PH3A    LNGNP    D-AR      1       3     NONE     -.3375     .1013  -3.33

EFFECTIVE NUMBER OF OBSERVATIONS . .           85
R-SQUARE . . . . . . . . . . . . . .        0.998
RESIDUAL STANDARD ERROR. . . . . . .  0.173031E-01
```

The second candidate model ARIMA(1,0,2) x ARIMA(0,1,1)$_4$ is specified and estimated below using the TSMODEL and ESTIMATE commands

➔ TSMODEL MODELB. MODEL IS @
LNGNP(4)=C2+(1,2 ; TH1B,TH2B)(4 ; TH4B)/(1; PH1B)NOISE.
➔ ESTIMATE MODELB. METHOD EXACT. HOLD RESIDUALS(RESB).

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODELB
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                         4
 LNGNP     RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-      VALUE      STD      T
   LABEL       NAME    DENOM.                 TRAINT                ERROR  VALUE
   1    C2             CNST      1      0     NONE      .0611      .0057  10.64
   2   TH1B    LNGNP    MA       1      1     NONE     -.3034      .1187  -2.56
   3   TH2B    LNGNP    MA       1      2     NONE     -.3434      .1106  -3.10
   4   TH4B    LNGNP    MA       2      4     NONE      .5607      .0930   6.03
   5   PH1B    LNGNP   D-AR      1      1     NONE      .7460      .0904   8.25

EFFECTIVE NUMBER OF OBSERVATIONS . .            87
R-SQUARE . . . . . . . . . . . . . .         0.998
RESIDUAL STANDARD ERROR. . . . . . .  0.175058E-01
```

The user can now perform diagnostic tests on the residual series, evaluate the two candidate models, and select the one that best fits the user's application.

The determination of differencing is handled very well in the RSFILTER command automatically and typically should be accepted by the user. However in some situations, the user may have special reasons to impose alternative differencing other than the differencing automatically determined by the RSFILTER command. This can be accomplished using the DFORDER and NODFORDER subcommands. For example, if we want to generate the R_t and S_t component series based on first-order and fourth-order differencing, the RSFILTER command is

➔ RSFILTER LNGNP. NEW-SERIES R, S. SEASONALITY 4. DFORDERS 1, 4.

If the user wants to exclude fourth-order differencing and impose first-order differencing, the following RSFILTER command can be specified

➔ RSFILTER LNGNP. NEW-SERIES R, S. SEASONALITY 4. @
DFORDERS 1. NODFORDER 4.

Other examples for the use of the optional DFORDER and NODFORDER subcommands can be found in the next chapter which describes the IARIMA command.

CHAPTER 5

ENHANCED EXPERT ARIMA MODELING

The SCA expert ARIMA modeling capabilities (IARIMA) are detailed in Chapter 2 of the SCA reference manual, *Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 2* along with related commands such as IESTIM. The other related command, RSFILTER, is discussed in the previous chapter of this document. In Release 7.0 (and above) of the SCA System, the capabilities of the IARIMA command have been further enhanced.

The algorithms and the rules for identifying ARIMA models using the IARIMA capability have been modified to perform more robustly. In addition, new options have been added to the IARIMA capability that provides more control to the user in final model determination. Specifically, options are now available to set the critical value for determining the statistical significance of parameter estimates retained in a model (through the CRITERIA subcommand), and for overriding differencing employed in the final model (through the DFORDER and NODFORDER subcommands).

The algorithm for the IARIMA command has been enhanced to better identify an ARIMA model when the selection between a pure AR and mixed ARMA model is marginal. For example, in earlier releases of the SCA System, the IARIMA command sometimes identified an AR(2) model when an ARMA(1,1) model may have been more appropriate. The SCA System now addresses such marginal cases more appropriately. Other improvements are made to the automatic model identification methods used in the IARIMA command on an ongoing basis. Modifications are typically made whenever users report data sets that prove to adversely affect general model identification.

The SEASONALITY (or PERIODICITY) subcommand in the IARIMA capability has been enhanced to search for seasonal patterns at very long lags. The previous release of the SCA System limited the seasonality cycle to 120 periods. Release 7.0 (and above) of the SCA System removes all practical limitations related to the identification of potential seasonality in a time series. For example, when considering hourly data, it is often important to include the 24 hour periodicity. Due to weekly patterns, it then becomes necessary to include a 168 periodicity in the model. The new release of the SCA System now allows for modeling high-order seasonal or periodic behaviors. However, when high-orders are used to handle higher frequency data, the exact maximum likelihood estimation method *cannot* be used. This is due to the fact that exact maximum likelihood estimation requires matrix inversion of high-order (e.g., 168 x 168). This will cause problems for matrix inversion in both accuracy and storage space. Therefore, conditional likelihood estimation must be used for such high-order models.

In previous releases of the SCA System, the DFORDER subcommand in the IARIMA command allowed the user to include specific differencing orders in an identified ARIMA model. However, if the IARIMA command found that the specified differencing order was not needed, it would exclude it from the final model. This behavior has been modified so that differencing orders specified in the DFORDER subcommand will always be retained in the final model.

Whereas the DFORDER subcommand allows the inclusion of specific differencing orders in the final model, it did not preclude the possibility that additional differencing orders may be found important and therefore added into the final model. For example, if a user specified a first-order difference using the DFORDER subcommand, the IARIMA command may also include a seasonal differencing term if the time series requires a seasonal differencing.

In the event a user wants to exclude specific differencing orders from the final model, Release 7.0 (and above) of the SCA System now includes a new subcommand, NODFORDER, that can be used in conjunction with the DFORDER subcommand to achieve full control of differencing orders in the automatic model identification of a time series.

If the NODFORDER subcommand is not specified, the IARIMA command will automatically determine whether any differencing orders are to be employed in the final model. Typically, if a differencing order is forced out of consideration through the NODFORDER subcommand, the IARIMA algorithm will compensate for the lack of differencing through one or more ARMA parameters.

The previous release of the IARIMA command used a critical value of 1.96 when determining whether a parameter estimate is statistically significant. A subcommand, CRITERIA, has been added to the IARIMA command to allow users more control over parameters included in the final model. This option is particularly useful when the time series to be modeled is either very long or rather short.

The IARIMA command uses the same approach of model identification as described for the RSFILTER command. However, the IARIMA command will provide the final identified model besides generating the nonseasonal and seasonal component series.

To illustrate the use of the IARIMA command, we use the LNGNP series introduced in the previous chapter to explain the RSFILTER command. The LNGNP series is the log transformed quarterly nominal gross national product (GNP, in billions of current dollars) of the United States between the first quarter of 1947 and the fourth quarter of 1969.

The IARIMA command requires the user to specify the potential seasonality of the time series being modeled (e.g., a monthly series would have a potential seasonality of 12, a daily series would have a potential seasonality of 7, and so on). Since the LNGNP series is a quarterly time series, it has a potential seasonality of 4. The following IARIMA command can be easily specified to automatically identify an appropriate model for the LNGNP series.

➔ IARIMA LNGNP. SEASONALITY 4.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- UTSMODEL
----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
         VARIABLE   OR CENTERED
                                        4
 LNGNP     RANDOM     ORIGINAL     (1-B  )
----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1                    CNST      1      0     NONE      .0606      .0041  14.96
  2           LNGNP     MA       1      4     NONE      .4306      .1167   3.69
  3           LNGNP    D-AR      1      1     NONE     1.0401      .1005  10.35
  4           LNGNP    D-AR      1      2     NONE      .0234      .1516    .15
  5           LNGNP    D-AR      1      3     NONE     -.3405      .1011  -3.37

TOTAL NUMBER OF OBSERVATIONS . . . .            92
EFFECTIVE NUMBER OF OBSERVATIONS . .            85
RESIDUAL STANDARD ERROR. . . . . . .  0.174545E
```

This model is good and can immediately be used for further application such as forecasting. By simply specifying the potential seasonality of the series, the IARIMA command automatically determines whether seasonal differencing and/or regular differencing is required to induce stationarity. In typical circumstances, the user should not override the differencing order(s) automatically determined by the IARIMA command unless there is good reason to do so. If the need to override differencing arises, the DFORDER and NODFORDER subcommands can be specified to force inclusion or exclusion of certain differencing order(s) in the final model similar to the RSFILTER command explained in a previous chapter.

In the above example, fourth-order differencing was automatically determined by the IARIMA command. If first-order differencing is also desired in the model, it can be forced into the final model by the following IARIMA command

➔ IARIMA LNGNP. SEASONALITY 4. DFORDER 1.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- UTSMODEL
----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
         VARIABLE   OR CENTERED
                                        4       1
 LNGNP     RANDOM     ORIGINAL     (1-B  ) (1-B  )
----------------------------------------------------------------------
 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1           LNGNP     MA       1      1     NONE     -.1982      .1050  -1.89
  2           LNGNP     MA       1      2     NONE     -.2522      .1068  -2.36
  3           LNGNP     MA       2      4     NONE      .5997      .0906   6.62

TOTAL NUMBER OF OBSERVATIONS . . . .            92
RESIDUAL STANDARD ERROR. . . . . . .  0.193055E-01
```

Note that by employing the "DFORDER 1" subcommand in the above IARIMA command, we effectively force first-order differencing in the model. Because IARIMA determined that seasonal differencing is also needed, the fourth-order differencing remains in the final model.

Whereas it is possible to force differencing by specifying the DFORDER subcommand, it is also possible to exclude differencing by specifying the NODFORDER subcommand. For example, if we had reason to force first-order differencing and exclude fourth-order differencing for the LNGNP series, the following command can be specified.

```
➔ IARIMA LNGNP.  SEASONALITY 4.  DFORDER 1.  NODFORDER 4.
```

```
THE CRITICAL VALUE FOR SIGNIFICANCE TESTS OF ACF AND ESTIMATES IS 1.875
SAMPLE ACF OF THE RESIDUALS (** SIGNIFICANT VALUES EXIST **)
 1 - 12    -.01  .02 -.12  .00 -.30 -.08 -.08  .07  .00  .11 -.00 -.07
T-VALUE    -.14  .15-1.09  .03-2.75 -.67 -.69  .56  .02  .94 -.04 -.60

SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- UTSMODEL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                         1
 LNGNP     RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
   1           LNGNP     MA       1      1     NONE     -.1703     .1031  -1.65
   2           LNGNP     MA       1      2     NONE     -.3072     .1066  -2.88
   3           LNGNP     MA       2      4     NONE      .6068     .0920   6.60
   4           LNGNP    D-AR      1      4     NONE      .9739     .0129  75.25

TOTAL NUMBER OF OBSERVATIONS . . . .            92
RESIDUAL STANDARD ERROR. . . . . . .  0.188898E-01
```

The IARIMA command (and RSFILTER command) does a very good job at determining appropriate differencing for the final model. Overriding the automatically determined differencing by alternative differencing is done at your own risk as it may result in over-differencing (or under-differencing) and produce less than optimal models. In the event the final model does not remove all significant autocorrelation from the residuals, the output from the IARIMA command informs the user that significant values exist in the autocorrelations of the residuals. This indicates that the model may have problems as demonstrated in the case of over differencing in the above example.

In practice, if the user is alerted to potential problems by significant residual autocorrelations, the user should check if potential seasonality is specified appropriately and then allow the IARIMA command to determine differencing automatically. Significant autocorrelations in the residual series may also be caused by other issues such as calendar effects, outliers, or the exclusion of important explanatory variables in the model. Such issues must be handled in an appropriate manner by studying the underlying causes rather than simply adding more ARMA parameters to the model.

CHAPTER 6

POWER TRANSFORMATION IN TIME SERIES FORECASTING

The Practitioner Edition (and above) provides the RETRANSFORM command to retransform forecasts back into original units when the forecasted series was transformed. It is important to note that a straight retransformation of forecasts produces biased forecasts. The RETRANSFORM command, based on the work of Guerrero (1993), adjusts for such bias. In this chapter, the RETRANSFORM command is presented. A few supporting SCA analytic statements are also used to complete the examples. More information on SCA analytic statements is documented in the *SCA Reference Manual for Fundamental Capabilities*. A related command, TSEARCH, which is used to search for the best power transformation for forecasting is described in Chapter 10.

Pankratz and Dudley (1987) and Guerrero (1993) developed a unified method to adjust for bias in retransforming forecasts. In the SCA System, we have implemented the RETRANSFORM command which is based on the method provided in Guerrero (1993). The capability provided in the RETRANSFORM command is not only appropriate for ARIMA model forecasts, it is also appropriate for forecasts generated by other models such as transfer function models, STF models, and vector ARMA models. Below is a brief summary of the method provided in Guerrero (1993).

In univariate time series analysis, Box and Jenkins (1976) discussed the feasibility of obtaining an ARIMA model with homogenous residual variance based on a time series derived from a power transformed series similar to that proposed in Box and Cox (1964). Using Y_t to denote the original series with positive value (if Y_t is non-positive, a positive constant value can be added to Y_t before the transformation), this class of transformation can be expressed as

$$Z_t = \begin{cases} (Y_t^{\lambda} - 1)/\lambda & \text{if } \lambda \neq 0 \\ \ell n(Y_t) & \text{if } \lambda = 0 \end{cases} \tag{6.1}$$

where Z_t is the transformed series, λ (referred to as lambda) is the power value of the transformation, and t=1, 2, …, n. The above formulation of power transformations is continuous in the parameter λ. While the transformation in (6.1) has desirable properties such as continuity of λ at 0, the variances of transformed series are not comparable when λ's are different. To address this issue, the following scaled power transformation is recommended by Box and Jenkins (1976), and Ansley, Spivey, and Wrobleski (1977)

$$Z_t = \begin{cases} (Y_t^{\lambda} - 1)/(\lambda \dot{Y}^{\lambda-1}) & \text{if } \lambda \neq 0 \\ \dot{Y} \ell n(Y_t) & \text{if } \lambda = 0 \end{cases} \tag{6.2}$$

where $\dot{Y} = (Y_1 \, Y_2 \, \cdots \, Y_n)^{1/n}$ is the geometric mean of the series. We shall refer to the formulation of transformation in (6.1) and (6.2) as Type 1 power transformations.

In addition to using the above power transformation, the following formulation of power transformation is also considered

$$Z_t = \begin{cases} Y_t^{\lambda} & \text{if } \lambda \neq 0 \\ \ell n(Y_t) & \text{if } \lambda = 0 \, . \end{cases} \tag{6.3}$$

This form of power transformation has a discontinuity at $\lambda = 0$, and may possess some other undesirable properties. However this form of power transformation is simple, and therefore is frequently used. We shall refer to this formulation of transformation as Type 2 power transformations.

There are two primary issues in the application of the power transformations shown above. The first is to select an appropriate lambda value that will either improve the efficacy of the model or the accuracy of the forecasts. This will be discussed in an upcoming chapter that documents the TSEARCH command. The second issue is the correction of biases induced by the retransformation on the forecasts. We will address the latter issue in this chapter.

6.1 Retransformation of Power Transformed Forecasts

When a transformation is applied to a time series, the forecasts are based on the transformed series and therefore must be retransformed back into the original metric. An issue with power transformation is that the straight retransformation of the forecasts is biased. This occurs because an estimated mean of a symmetric distribution in the transformed data becomes an estimated median after retransformation of the estimate. A number of research papers have attempted to address this issue. Pankratz and Dudley (1987) and Guerrero (1993) developed a unified method to adjust for the bias in retransforming forecasts. Below is a summary of the method provided in Guerrero (1993).

Unbiased Retransformation

The power transformation described in (6.1), (6.2) and (6.3) can be generally expressed as

$$Z_t = T(Y_t) \tag{6.4}$$

where $T(\bullet)$ is the transformation operator, and $T^{-1}(\bullet)$ is the inverse-transformation operator. Thus, we have

$$Y_t = T^{-1}(Z_t) \, . \tag{6.5}$$

Assuming that $\hat{Z}_n(\ell)$ is the ℓ-step-ahead MMSE forecast of $Z_{n+\ell}$ at the forecast origin $t = n$, the straight retransformation of $\hat{Z}_n(\ell)$ under (6.1) is

$$\hat{Y}_n(\ell) = T^{-1}(\hat{Z}_n(\ell)) = \begin{cases} (\lambda \hat{Z}_n(\ell) + 1)^{1/\lambda} & \text{if } \lambda \neq 0 \\ e^{\hat{Z}_n(\ell)} & \text{if } \lambda = 0 . \end{cases} \tag{6.6}$$

It has been shown that $\hat{Y}_n(\ell)$ is biased. To correct for the bias, Guerrero (1993) derives the following unbiased estimate $\tilde{Y}_n(\ell)$ where

$$\tilde{Y}_n(\ell) = T^{-1}(\hat{Z}_n(\ell)) \cdot C_\lambda(\ell) . \tag{6.7}$$

The $C_\lambda(\ell)$ is the debiasing factor which can be expressed as

$$C_\lambda(\ell) = \begin{cases} \left\{ 0.5 + 0.5\left[1 + 2\left(\lambda^{-1} - 1\right) R_\ell^2 \right]^{1/2} \right\}^{1/\lambda}, \quad R_\ell = \dfrac{\sigma_\ell}{\lambda^{-1} + \hat{Z}_n(\ell)} & \text{if } \lambda \neq 0 \\ e^{\sigma_\ell^2 / 2} & \text{if } \lambda = 0 \end{cases} \tag{6.8}$$

and σ_ℓ is the ℓ-step-ahead forecast standard error of $\hat{Z}_n(\ell)$. It is readily seen that the debiasing factor is a function of the transformed forecast and its standard error if $\lambda \neq 0$, and is a function of forecast standard error only if $\lambda = 0$.

The forecast limits of $Z_{n+\ell}$ can be expressed as

$$\hat{Z}_n(\ell) \pm z_{\alpha/2} \cdot \sigma_\ell . \tag{6.9}$$

The straight retransformation of the above forecast limits $T^{-1}\left(\hat{Z}_n(\ell) \pm z_{\alpha/2} \cdot \sigma_\ell\right)$ is also biased. To correct for the bias, Guerrero (1993) suggests multiplying the forecast limits by their respective debiasing factors.

The unbiased retransformation of the forecasts under (6.2) and (6.3) can be derived similarly. In the case of (6.2), the straight retransformation of $\hat{Z}_n(\ell)$ is

$$\hat{Y}_n(\ell) = T^{-1}\left(\hat{Z}_n(\ell)\right) = \begin{cases} \left(\lambda\left(\hat{Z}_n(\ell) \dot{Y}^{\lambda-1}\right) + 1\right)^{1/\lambda} & \text{if } \lambda \neq 0 \\ e^{\hat{Z}_n(\ell)} & \text{if } \lambda = 0 . \end{cases} \tag{6.10}$$

Again the above retransformed forecast $\hat{Y}_n(\ell)$ is biased. Using equation (6.7), the unbiased forecast $\tilde{Y}_n(\ell)$ can be obtained by using the following debiasing factor $C_\lambda(\ell)$

$$C_\lambda(\ell)=\begin{cases}\left\{0.5+0.5\left[1+2\left(\lambda^{-1}-1\right)R_\ell^2\right]^{1/2}\right\}^{1/\lambda}, \quad R_\ell=\dfrac{\sigma_\ell \dot{Y}^{\lambda-1}}{\lambda^{-1}+\hat{Z}_n(\ell)\dot{Y}^{\lambda-1}} & \text{if } \lambda\neq 0\\ e^{\left(\sigma_\ell/\dot{Y}\right)^2/2} & \text{if } \lambda=0\,.\end{cases} \tag{6.11}$$

For the class of power transformation in (6.3) which we refer to as Type 2 power transformations, the straight retransformation of $\hat{Z}_n(\ell)$ is

$$\hat{Y}_n(\ell)=T^{-1}\left(\hat{Z}_n(\ell)\right)=\begin{cases}\hat{Z}_n(\ell)^{1/\lambda} & \text{if } \lambda\neq 0\\ e^{\hat{Z}_n(\ell)} & \text{if } \lambda=0\end{cases} \tag{6.12}$$

which is also biased. Using equation (6.7), the unbiased forecast $\tilde{Y}_n(\ell)$ can be obtained by using the following debiasing factor $C_\lambda(\ell)$

$$C_\lambda(\ell)=\begin{cases}\left\{0.5+0.5\left[1+2\left(\lambda^{-1}-1\right)R_\ell^2\right]^{1/2}\right\}^{1/\lambda}, \quad R_\ell=\dfrac{\sigma_\ell}{\hat{Z}_n(\ell)} & \text{if } \lambda\neq 0\\ e^{\sigma_\ell^2/2} & \text{if } \lambda=0\,.\end{cases} \tag{6.13}$$

6.2 U.S. GNP Example of Retransformation

In previous chapters of the document, the log transformed quarterly nominal gross national product of the United States (LNGNP) was used to illustrate the RSFILTER and IARIMA commands. The LNGNP series shall be used to illustrate the retransformation of forecasts back into original units. Using the model identified by the IARIMA command, the model is re-specified below using the TSMODEL command and estimated using the ESTIMATE command. The FORECAST command is then specified to generate eight forecasts from the end of the LNGNP series

```
➔ TSMODEL UTSMODEL.  MODEL LNGNP(4)=C+(4)/(1 TO 3)NOISE.
➔ ESTIMATE UTSMODEL.  METHOD EXACT.
➔ FORECAST UTSMODEL.  NOFS 8.  HOLD FORECAST(FLN), STDERR(FSELN).
```

```
 ---------------------------------
   8 FORECASTS, BEGINNING AT   92
 ---------------------------------
  TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
    93      5.4366      0.0175
    94      5.4934      0.0252
    95      5.4930      0.0317
    96      5.5490      0.0349
    97      5.4822      0.0399
    98      5.5469      0.0422
    99      5.5524      0.0434
   100      5.6133      0.0436
```

The RETRANSFORM command is used to retransform the forecasts back into original units. The unbiased method is employed. However, it is necessary to specify $\lambda = 0$ (indicating that a logarithmic transformation was applied to the original series) and TYPE=1 (indicating Type 1 transformation was applied)

```
➔ VLAMBDA=0.0
➔ VTYPE=1
➔ RETRANSFORM FLN, FSELN.  METHOD UNBIASED. @
      FORM VLAMBDA, VTYPE.
```

```
TIME     FORECAST    STD. ERROR  RETRANSFORMED
  1         5.437         0.017       229.751
  2         5.494         0.025       243.287
  3         5.494         0.032       243.332
  4         5.550         0.035       257.363
  5         5.483         0.040       240.833
  6         5.548         0.042       256.983
  7         5.554         0.043       258.452
  8         5.614         0.043       274.618
```

For comparison purposes, the following straight retransformation of the forecasts is presented.

```
➔ RETRANSFORM FLN, FSELN.  METHOD STRAIGHT. @
      FORM VLAMBDA, VTYPE.
```

```
TIME     FORECAST    STD. ERROR  RETRANSFORMED
  1         5.437         0.017       229.716
  2         5.494         0.025       243.211
  3         5.494         0.032       243.211
  4         5.550         0.035       257.209
  5         5.483         0.040       240.644
  6         5.548         0.042       256.755
  7         5.554         0.043       258.210
  8         5.614         0.043       274.358
```

As shown in the above table, the unbiased and straight retransformed forecasts for this series are somewhat different, with the unbiased forecasts slightly larger than the straight retransformed forecasts.

The natural logarithmic transformation is often used in practice because of its desirable properties and the ease of interpretation when used for business and econometric applications. However, in some situations we may wish to investigate alternative power transformations that may be better suited for the time series under study. Such capability is provided by the TSEARCH command.

CHAPTER 7

CAUSALITY TESTING

The Professional Edition (A) of the SCA Statistical System implements new capabilities for causality testing following the work of Chen and Lee (1990) and others. In this chapter, the CAUSALTEST command is presented. Related commands including MTSMODEL and MESTIM are documented in the SCA reference manual, *Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 2*.

Chen and Lee (1990) used vector ARMA (VARMA) models for causality testing. The traditional hypothesis testing procedure provides a framework to contrast a null hypothesis versus an alternative hypothesis. Without imposing *a priori* restrictions, an empirical study of dynamic relationships often involves several non-nested hypotheses. Consequently, a more systematic approach is required to examine the multiple hypotheses so that the test conclusion is not affected by *a priori* choice of the alternative. The bivariate VARMA test developed by Chen and Lee (1990) addresses these issues more adequately and is the focus of causality testing discussed in this chapter.

7.1 Causality Testing Using Vector ARMA Models

A number of time series models can be employed for causality testing (see e.g., Sims, 1972; and Haugh, 1976). Because vector ARMA (VARMA) models have been shown to be effective in forecasting, this class of models can also be used for causality testing (see e.g., Pierce and Haugh, 1977; Kang, 1981; Koreisha, 1983; and Chen and Lee, 1990). Using conventional notations, a bivariate VARMA(p,q) model can be expressed as

$$(\mathbf{I}-\phi_1 B-\cdots-\phi_p B^p)\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \mathbf{C} + (\mathbf{I}-\theta_1 B-\cdots-\theta_q B^q)\begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \qquad (7.1)$$

where ϕ_i's and θ_j's are 2×2 matrices, **C** is a 2×1 constant vector, and $\mathbf{a}_t = [a_{1t},\ a_{2t}]'$ is a sequence of 2×1 random shock vectors identically and independently distributed as a normal distribution with zero mean and covariance matrix Σ with $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$. For convenience, the model in (7.1) can be re-written as

$$\begin{bmatrix} \phi_{11}(B) & \phi_{12}(B) \\ \phi_{21}(B) & \phi_{22}(B) \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \mathbf{C} + \begin{bmatrix} \theta_{11}(B) & \theta_{12}(B) \\ \theta_{21}(B) & \theta_{22}(B) \end{bmatrix} \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \qquad (7.2)$$

where $\phi_{ij}(B) = \phi_{ij0} - \phi_{ij1}B - \phi_{ij2}B^2 - \cdots$, and $\theta_{ij}(B) = \theta_{ij0} - \theta_{ij1}B - \theta_{ij2}B^2 - \cdots$. It is important to note that $\phi_{ij0} = \theta_{ij0} = 1$ if $i = j$, and $\phi_{ij0} = \theta_{ij0} = 0$ if $i \neq j$.

Assuming the form of the model in (7.2) is known, sufficient conditions for testing the hypotheses H_1, H_2, H_3, H_4, and H_5 using $\phi_{ij}(B)$ and $\theta_{ij}(B)$ of equation (7.2) are listed below:

Hypothesis	Sufficient Conditions (Constraints)	
$H_1 :\ Y \wedge X$	$\phi_{12}(B) = \phi_{21}(B) = 0,\ \ \theta_{12}(B) = \theta_{21}(B) = 0,\ \ \sigma_{12} = \sigma_{21} = 0$.	
$H_2 :\ Y \leftrightarrow X$	$\phi_{12}(B) = \phi_{21}(B) = 0,\ \ \theta_{12}(B) = \theta_{21}(B) = 0$.	
$H_3 :\ Y \nLeftarrow X$	$\phi_{12}(B) = \theta_{12}(B) = 0$	(7.3)
$H_4 :\ Y \nRightarrow X$	$\phi_{21}(B) = \theta_{21}(B) = 0$	
$H_5 :\ Y \Leftrightarrow X$	no constraints	

The conditions in (7.3) become necessary and sufficient conditions if the model in (7.2) is a pure vector AR or a pure vector MA model. In the above hypotheses, H_3 implies the past X does not help to predict future Y, and H_4 implies the past Y does not help to predict X. In both situations, we assume σ_{12} may be nonzero. However, if σ_{12} equals to zero, the hypotheses H_3, H_4, and H_5 can be tested under a more stringent condition. Therefore the following three additional hypotheses should also be considered:

Hypothesis	Sufficient Conditions (Constraints)	
$H_3^* :\ Y <\!\nLeftarrow X$	$\phi_{12}(B) = \theta_{12}(B) = 0,\ \ \sigma_{12} = 0$	
$H_4^* :\ Y \nRightarrow\!> X$	$\phi_{21}(B) = \theta_{21}(B) = 0,\ \ \sigma_{12} = 0$	(7.4)
$H_5^* :\ Y <\!\Leftrightarrow\!> X$	$\sigma_{12} = 0$	

In the above hypotheses, H_3^* implies both past and concurrent X do not help to predict Y, and H_4^* implies both past and concurrent Y do not help to predict X. For H_5^*, it implies a "true" feedback relationship since Y and X are not contemporaneously related.

7.2 A Decision Tree Approach for Detecting Dynamic Relationships

It is important to note that the eight hypotheses stated in (7.3) and (7.4) are neither nested nor mutually exclusive to each other. This situation leads to significant complications in conducting appropriate hypothesis testing. In the causality testing literature (see e.g., Sims, 1972; Haugh, 1976; Pierce, 1977; Feige and Pearce, 1979; and Hsiao, 1979), most tests are designed to discriminate between independency and an alternative hypothesis. The primary purpose of these tests is to reject the independency (so called "neutrality") hypothesis. However, it is more useful to identify the true nature of the relationship between two series. To accomplish this goal, Chen and Lee (1990) proposed a decision tree approach which consists of testing a sequence of pair-wise hypotheses that are defined by each of the above relationships. This inference procedure is based on the principle that a maintained hypothesis should not be rejected unless there is sufficient evidence against it.

Two procedures for identifying dynamic relationships are considered here: (1) the backward procedure and (2) the forward procedure. The backward procedure takes the position that a hypothesis should not be rejected in favor of a more restrictive one unless sufficient evidence indicates otherwise. Consequently, the statistical procedure starts from the most general hypothesis, H_5, and then examines the relative validity of competing hypotheses in an increasing order of parameter restrictions. On the other hand, the forward procedure asserts that a simpler model is preferred unless the evidence strongly suggests otherwise. Hence, the forward procedure starts its test from the most restrictive hypothesis, H_1, and moves toward less restrictive hypotheses. In both procedures, each step of the test examines one or two pairs of nested hypotheses. Typically, both testing procedures result in the same conclusion. In some marginal situations, the backward procedure tends to favor more complicated relationships while the forward procedure tends to favor simpler relationships.

Generally speaking, the forward procedure works better (i.e., the test procedure has higher discriminating power) if the variables considered are likely to be independent or have a more restrictive relationship. On the other hand, the backward procedure works better if the variables considered are likely to have more complex relationships. Since we are more interested in the latter situation, the backward procedure is discussed first. The flow charts for both testing procedures are included in the following discussion.

The Backward Procedure

The first step of backward procedure, B1, is to examine two pairs of hypotheses: (a) H_3 vs H_5 and (b) H_4 vs H_5. This step, distinguishing the feedback relationship from unidirectional relationship, gives rise to four possible outcomes, E_1 to E_4, as follows:

E_1: H_3 is not rejected in the pair-wise test (a) and H_4 is rejected in the pair-wise test (b),

E_2: H_3 is rejected in test (a) and H_4 is not rejected in test (b),

E_3: H_3 is not rejected in test (a) and H_4 is not rejected in (b),

E_4: H_3 is rejected in test (a) and H_4 is rejected in test (b).

The outcome of E_1 implies that the past information of Y may help to predict current X, but the past X does not help to predict current Y. Hence, this outcome leads to the next pair-wise test (g), H_3^* vs H_3, where we try to detect the contemporaneous effect in the unidirectional relationship. If H_3^* is rejected in test (g), the conclusion, $Y \Rightarrow X$, is reached; otherwise the conclusion, $Y \Rightarrow> X$, would be made. Similarly, the occurrence of events E_2 and E_4, respectively, suggests a possible unidirectional relationship from X to Y and a possible feedback relationship between Y and X. Therefore, the outcome of E_2 leads to the pair-wise test (h), which helps us to choose between H_4^* and H_4. Under the outcome of E_4, it requires the test (i) which discriminates between the strong feedback hypothesis

(H_5^*) and the weak feedback hypothesis (H_5). The rejection of H_4^* in test (h) implies $Y \Leftarrow X$. Otherwise, the conclusion, $Y <\Leftarrow X$, would be reached. In test (i), the rejection of H_5^* implies $Y \Leftrightarrow X$. If H_5^* is not rejected, we can conclude $Y <\Leftrightarrow> X$.

When one of the events, E_1, E_2, and E_4, occurs in sequence B1, the backward procedure stops at the end of test (g), test (h), and test (i) respectively. If neither H_3 nor H_4 is rejected, (i.e., E_3 is realized), the backward procedure will move to sequence B2 where two pairs of hypotheses will be examined: (c) H_2 vs H_3 and (d) H_2 vs H_4. Again, four possible results may come out of this sequence. They are summarized as follows:

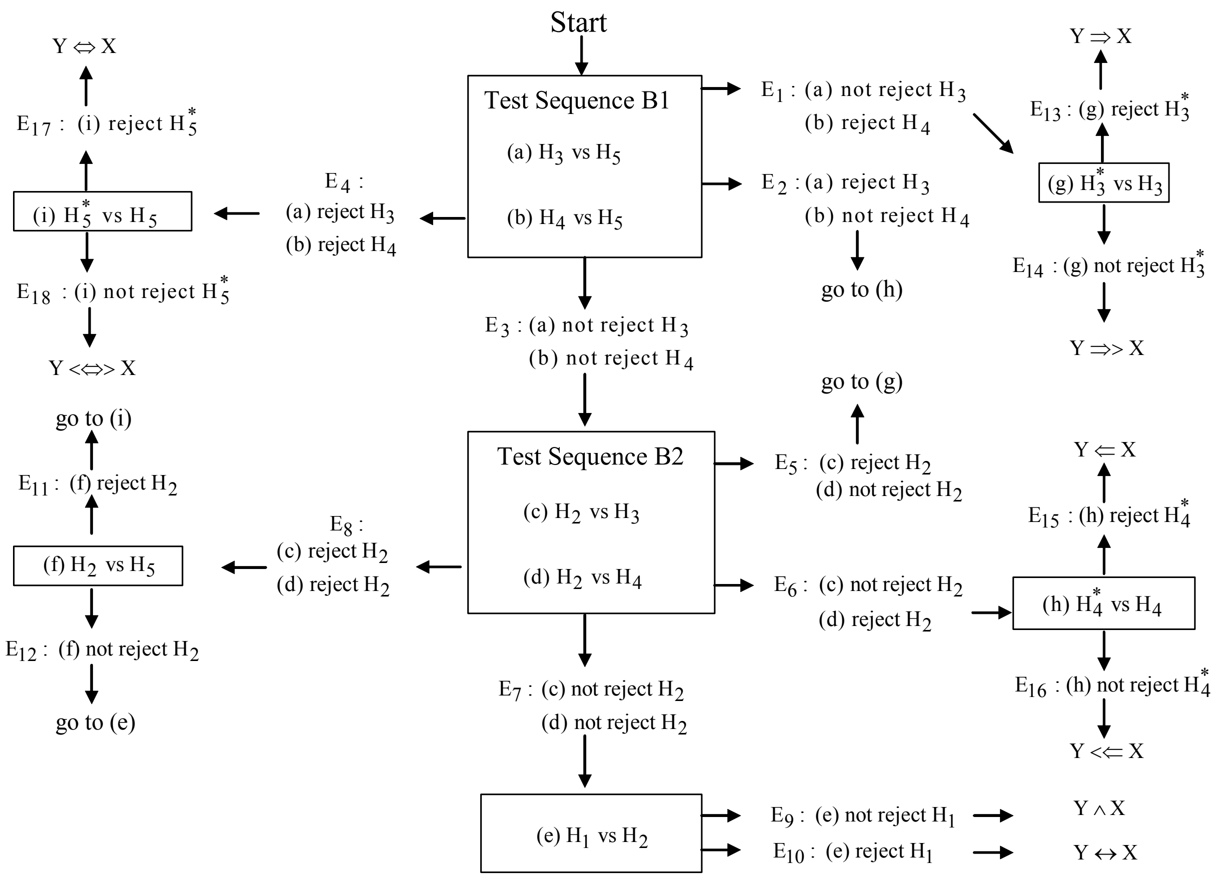
E_5: H_2 is rejected in pair-wise test (c) but is not rejected in test (d),

E_6: H_2 is not rejected in test (c) but is rejected in test (d),

E_7: H_2 is rejected in either test (c) or test (d),

E_8: H_2 is rejected in both test (c) and test (d).

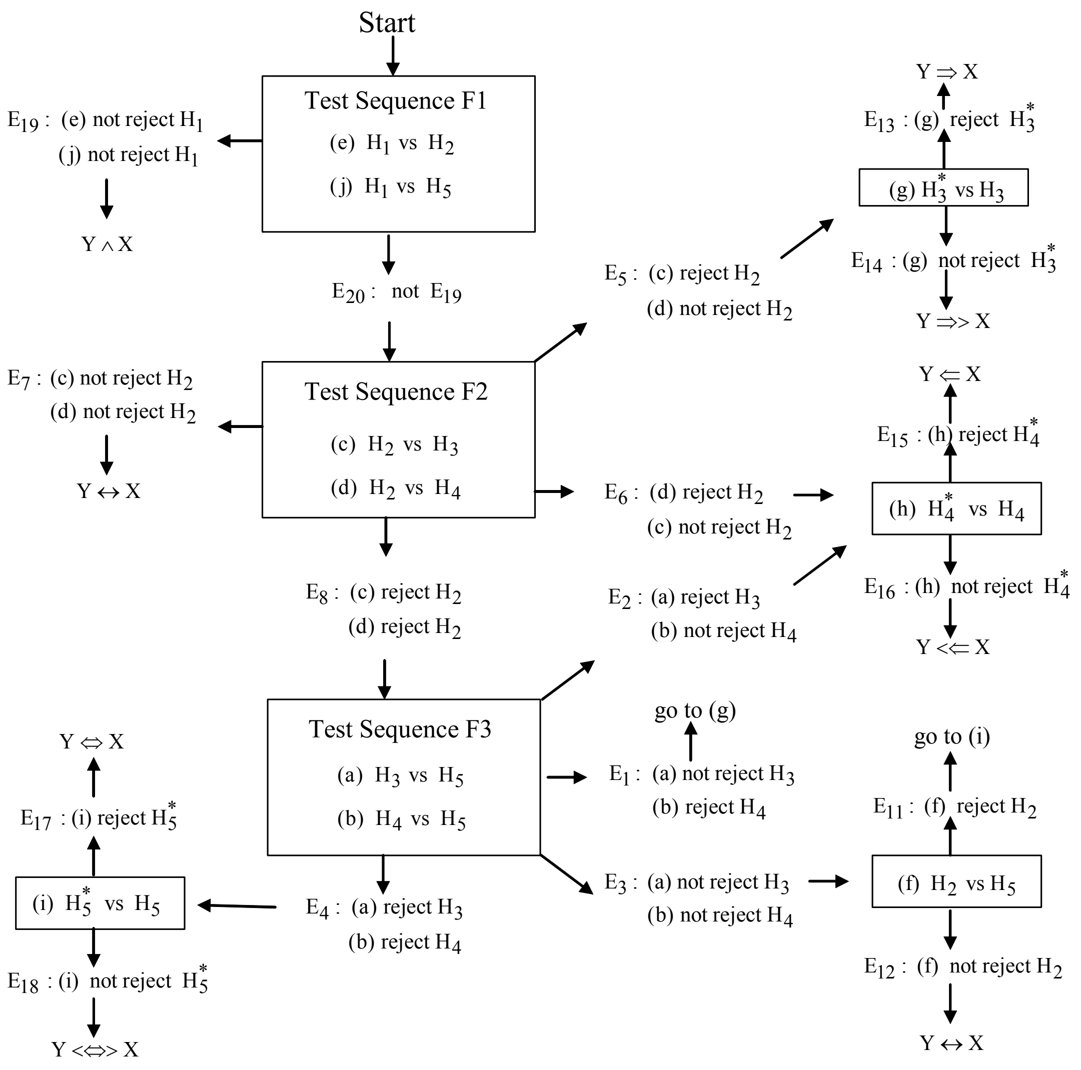
Since test (c) examines the possibility of $Y \Rightarrow X$ and test (d) examines that of $Y \Leftarrow X$, outcome E_5 implies that the relationship $Y \Rightarrow X$ is more probable than $Y \Leftarrow X$. Therefore, the result of event E_5 leads to test (g). A similar argument suggests that the occurrence of E_6 leads to test (h). A definitive conclusion will be reached at the end of tests (g) and (h). The rejection of H_2 in both test (c) and test (d) indicate the equal possibility of $Y \Leftarrow X$ and $Y \Rightarrow X$. Hence, the result of E_8 calls for test (f): H_2 versus H_5. If H_2 is rejected at test (f), then the possibility of the feedback relationship is established and the backward procedure moves to test (i). When H_2 is not rejected at test (f) or when event E_7 is realized, the backward procedure then proceeds to test (e), which discriminates between the independency and the contemporaneous relationship. If H_1 is rejected in test (e), the conclusion of $Y \leftrightarrow X$ is reached. Otherwise, $Y \wedge X$ will be the case. In each step of the hypothesis testing, the likelihood ratio test is conducted. A flow chart representation of the backward procedure is displayed below.



The Forward Procedure

The forward procedure begins by testing the validity of the independency hypothesis at sequence F1. The hypothesis indices, H_1 to H_5, the outcome indices, E_1 to E_8, and the pair-wise test indices, (a) to (h), are consistent in flow chart of the backward procedure shown above, and the flow chart of the forward procedure that follows. The sequence F1 considers two pairs of hypotheses testing, test (e) and test (j). If H_1 is not rejected in either test, the conclusion of $Y \wedge X$ is reached and the forward procedure stops. Otherwise, the procedure will move forward to sequence F2, which examines the relative likelihood of the contemporaneous relationship versus the unidirectional relationship. Notice that sequence F2 is identical to sequence B2, where one of the four possible outcomes, E_5, E_6, E_7, and E_8, will emerge. Using the same argument on sequence B2, the outcomes of E_5 and E_6 lead to tests (g) and (h) respectively. A conclusion from one of the four possible unidirectional relationships can be reached as a result and the forward procedure stops. The outcome of E_7 implies $Y \leftrightarrow X$ and stops the forward procedure. However, the outcome of E_8, which rules out the case of a contemporaneous relationship, leads the forward procedure to sequence F3, which corresponds to sequence B1 in the backward procedure. Tests (a) and (b) may generate one of four possible outcomes, E_1, E_2, E_3, and E_4. Similar to sequence B1 in the backward procedure, the outcomes of

E_1 and E_2 lead to tests (g) and (h) respectively. One of the four unidirectional relationships will be detected as a result and the procedure stops. The outcome of E_4 implies a possible feedback relationship, and a further study, test (i), is needed to identify its nature. When H_5^* is rejected in test (i), we conclude $Y \Leftrightarrow X$; otherwise, we conclude $Y <\Leftrightarrow> X$. The outcome of E_3 implies that Y may help to predict X and X may help to predict Y, but the nature of this dynamic relationship is not clear. Therefore, test (f) is needed. When H_2 is not rejected in test (f), the conclusion $Y \leftrightarrow X$ is reached and the procedure stops. If H_2 is rejected in test (f), the procedure moves to test (i) to determine the nature of the feedback relationship. Consequently, either $Y \Leftrightarrow X$ or $Y <\Leftrightarrow> X$ is shown to exist. A flow chart representation of the forward procedure is presented below.



7.3 Test Statistic for Each Pair-wise Test

In practice, the model(s) for the time series under study are unknown. However, the order of the vector ARMA model for the series can be determined using the model identification procedure discussed in Chapter 14 of Liu (2006). The test procedures are rather robust with respect to the selected model as long as the order of the model is generally correct. Corresponding to each hypothesis, the parameters of the constrained model can be estimated using the maximum likelihood estimation method discussed in Liu (2006) (also see Hillmer and Tiao, 1979). The likelihood ratio statistic is then calculated for each pair of hypotheses:

$$\mathrm{LR}(\mathrm{H_i}\ \text{vs}\ \mathrm{H_j}) = \ell(\mathrm{H_i}) - \ell(\mathrm{H_j}) \tag{7.5}$$

where $\ell(\mathrm{H_i}) = -2 *$ (log of the maximum likelihood value under $\mathrm{H_i}$). The above likelihood ratio statistic follows a χ^2-distribution with ν degrees of freedom where ν in each test is the difference between the numbers of estimated parameters under the null (the more restrictive one) and the alternative (the less restrictive one) hypotheses. A chi-square table can then be used to determine the significance of the test statistic for the tested hypotheses.

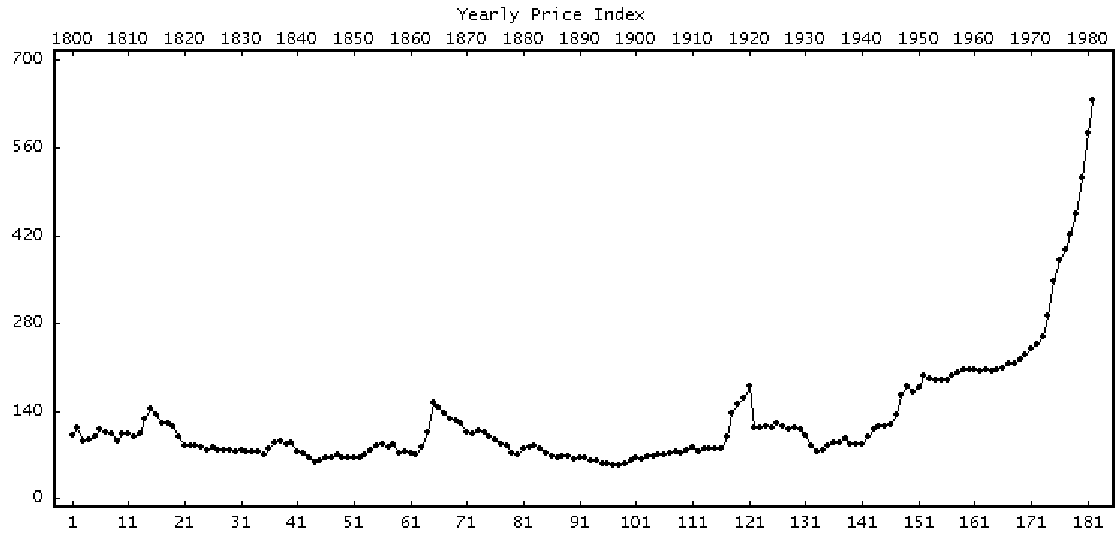
In each procedure, an α significance level will be used in conducting all pair-wise tests. Note that this α level is not the Type I error probability for the overall performance of the procedures. It serves only as a cut-off point in a sequential decision procedure. The smaller the α, the higher is the probability that the more restrictive hypothesis will not be rejected. Hence, taking a smaller α is equivalent to favoring the more restrictive hypotheses (i.e., simpler relationships), and taking a larger α is equivalent to favoring the more complicated relationships.

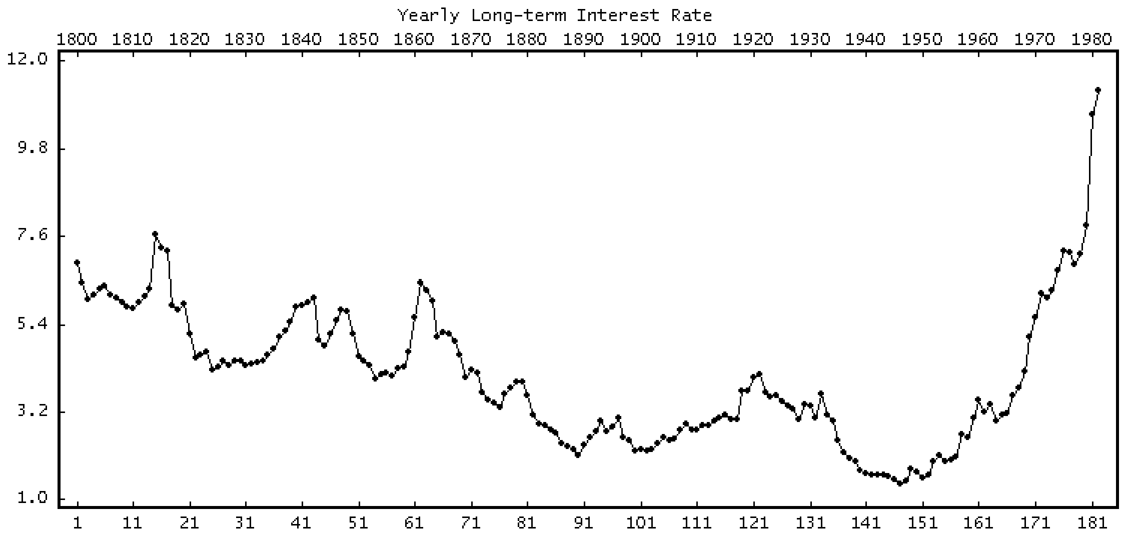
We shall employ two actual examples to illustrate causality testing using vector ARMA models. The first example is the United States yearly price index and long-term interest rate. This data set is used in various studies of the Gibson paradox. The second example is the non-seasonally adjusted monthly housing market data in the United States. The housing starts and houses sold series are employed in this study. Additional examples can be found in Liu (2006).

7.4 Causal Relationships between Price and Interest Rate in the United States

The United States yearly price index and interest rate between 1800 and 1981 are shown below using the SCAGRAF capability.

U.S. yearly price index and long-term interest rate (1800-1981)





In the above figure, we observe the striking positive association between the price level and the interest rate. Such a relationship has been repetitively confirmed by various statistical methods ranging from correlation to spectral analysis, and is considered to be an established fact (see e.g., Fisher, 1907; Wicksell, 1907; Keynes, 1930; Sargent, 1973; Shiller and Siegel, 1977; Lee and Petruzzi, 1986; and Barsky and Summers, 1988). Keynes (1930) called this association the "Gibson paradox" due to conflicting conclusions on this phenomenon when various statistical methods are used. Nelson and Schwertz (1982) compare several methods including usual correlation, pre-whitening, two-sided regression, and vector ARMA models by simulation studies. They conclude that vector ARMA models provide the most powerful test for the presence of Granger causality.

Chen and Lee (1990) also employ vector ARMA models to examine the dynamic relationships between prices and interest rates in United States. The dynamic relationships revealed by vector ARMA models provide economists new insight in discriminating among competing theories about the nature of the Gibson paradox derived from the data. Instead of using the entire series, the non-gold standard period between 1914 and 1981 are used in this example and logarithmic transformation is applied to both series for the convenience of interpretation. The series Y_t and X_t in the following discussions are defined as

Y_t : log transformed United States yearly price index between 1914 and 1981, and
X_t : log transformed United States yearly interest rate between 1914 and 1981.

During this sub-period, we have 68 observations for each series.

Following vector ARMA model identification procedures, we find that a vector ARMA(1,1) model of the form

$$(\mathbf{I}-\phi B)\begin{bmatrix} Y_t \\ X_t \end{bmatrix} = \mathbf{C} + (\mathbf{I}-\theta B)\mathbf{a}_t \quad . \tag{7.6}$$

is appropriate for these two series. The model is specified using the MTSMODEL command

```
➔ MTSMODEL  ARMA11. SERIES ARE LUSP,LUSR.            @
           MODEL IS (1-PHI*B)SERIES=C+(1-TH1*B)NOISE.
```

After the vector ARMA(1,1) model is specified, the causality test is run using the CAUSALTEST command

```
➔ CAUSALTEST MODEL ARMA11.  OUTPUT PRINT(CORR)
```

```
 ----------------------
  ERROR COVARIANCE MATRIX
  ----------------------
               1          2
   1      .008147
   2     -.000286     .006977
```

```
==============================================================
SUMMARY OF FINAL PARAMETER ESTIMATES AND THEIR STANDARD ERRORS
==============================================================

PARAMETER             PARAMETER                  FINAL         ESTIMATED
 NUMBER              DESCRIPTION               ESTIMATE       STD.  ERROR
----------    -----------------------------     ------------    ------------
    1               CONSTANT( 1)               -0.006581        0.167474
    2               CONSTANT( 2)               -0.460388        0.117567
    3         AUTOREGRESSIVE ( 1, 1, 1)         1.004013        0.037627
    4         AUTOREGRESSIVE ( 1, 1, 2)         0.015832        0.039399
    5         AUTOREGRESSIVE ( 1, 2, 1)         0.100840        0.026400
    6         AUTOREGRESSIVE ( 1, 2, 2)         0.971504        0.027438
    7         MOVING AVERAGE ( 1, 1, 1)        -0.312030        0.114152
    8         MOVING AVERAGE ( 1, 1, 2)         0.196522        0.125885
    9         MOVING AVERAGE ( 1, 2, 1)        -0.279190        0.113242
   10         MOVING AVERAGE ( 1, 2, 2)         0.048165        0.123562

CAUSALITY TEST BETWEEN VARIABLES   LUSP   AND   LUSR

-2*(LOG LIKELIHOOD) UNDER H1,H2,H3,H3*,H4,H4*,H5,H5* ARE:
 -0.49779553E+03
 -0.49780307E+03
 -0.51833948E+03
 -0.51828082E+03
 -0.49996078E+03
 -0.49990012E+03
 -0.52080267E+03
 -0.52071661E+03

RESULT BASED ON THE BACKWARD PROCEDURE ( Y:LUSP    ,  X: LUSR    )
    LUSP =>> LUSR     (Y STRONGLY CAUSES X)

RESULT BASED ON THE FORWARD PROCEDURE  ( Y:LUSP    ,  X: LUSR    )
    LUSP =>> LUSR     (Y STRONGLY CAUSES X)
```

Based on the above output, the values of $\ell(H_i)$ under various hypotheses are

$$\begin{aligned}
\ell(H_1) &= -497.796\\
\ell(H_2) &= -497.803\\
\ell(H_3) &= -518.339\\
\ell(H_3^*) &= -518.281\\
\ell(H_4) &= -499.961\\
\ell(H_4^*) &= -499.900\\
\ell(H_5) &= -520.803\\
\ell(H_5^*) &= -520.717\ .
\end{aligned} \tag{7.7}$$

Summarizing the SCA output of the log likelihood ratio statistics for various pairs of hypotheses and their corresponding χ^2 critical values at $\alpha = 5\%$, the following table is constructed:

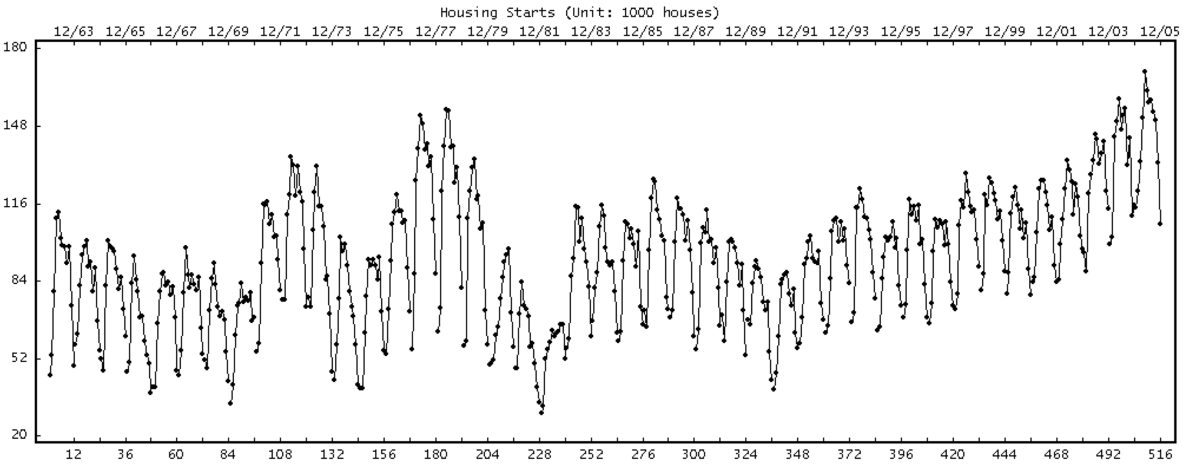
$$\begin{array}{llll}
LR(H_3 \text{ vs } H_5) = \ell(H_3) - \ell(H_5) = \ 2.463 & < \chi^2_{0.05}(2) = 5.991 & \text{(insignificant)} \\
LR(H_4 \text{ vs } H_5) = \ell(H_4) - \ell(H_5) = 20.842 & > \chi^2_{0.05}(2) = 5.991 & \text{(significant)} \\
LR(H_2 \text{ vs } H_3) = \ell(H_2) - \ell(H_3) = 20.534 & > \chi^2_{0.05}(2) = 5.991 & \text{(significant)} \\
LR(H_2 \text{ vs } H_4) = \ell(H_2) - \ell(H_4) = \ 2.158 & < \chi^2_{0.05}(2) = 5.991 & \text{(insignificant)} \\
LR(H_1 \text{ vs } H_2) = \ell(H_1) - \ell(H_2) = \ 0.008 & < \chi^2_{0.05}(1) = 3.841 & \text{(insignificant)} \\
LR(H_2 \text{ vs } H_5) = \ell(H_2) - \ell(H_5) = 22.996 & > \chi^2_{0.05}(4) = 9.488 & \text{(significant)} \\
LR(H_3^* \text{ vs } H_3) = \ell(H_3^*) - \ell(H_3) = \ 0.059 & < \chi^2_{0.05}(1) = 3.841 & \text{(insignificant)} \\
LR(H_4^* \text{ vs } H_4) = \ell(H_4^*) - \ell(H_4) = \ 0.061 & < \chi^2_{0.05}(1) = 3.841 & \text{(insignificant)} \\
LR(H_5^* \text{ vs } H_5) = \ell(H_5^*) - \ell(H_5) = \ 0.086 & < \chi^2_{0.05}(1) = 3.841 & \text{(insignificant)} \\
LR(H_1 \text{ vs } H_5) = \ell(H_1) - \ell(H_5) = \ 23.007 & > \chi^2_{0.05}(5) = 11.070 & \text{(significant)} .
\end{array} \tag{7.8}$$

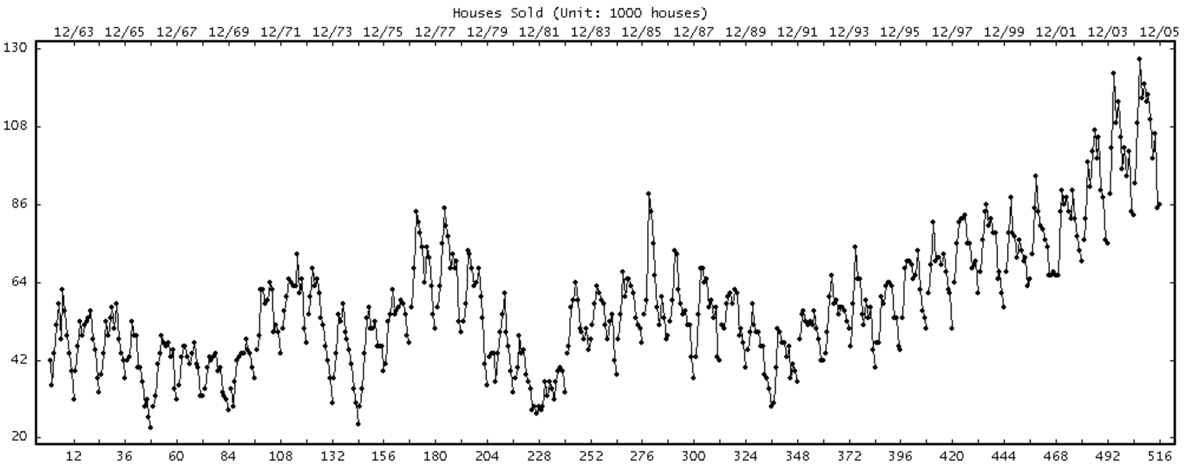
Using the above likelihood ratio statistics and both the backward and forward procedures suggested by Chen and Lee (1990), the CAUSALTEST concludes is that price strongly causes interest rates as shown in the output of the CAUSALTEST command. More information on the backward and forward procedures used in CAUSALTEST can be found in Chen and Lee (1990) and Liu (2006).

7.5 Causal Relationships between Housing Starts and Houses Sold in the United States (Based on Non-Seasonally Adjusted Series)

In this example, we consider non-seasonally adjusted (NSA) U.S. monthly single-family housing starts and houses sold. The monthly data for these series between January 1963 and December 2003 are shown below.

U.S. monthly single-family housing starts and houses sold (NSA, 1/1963 – 12/2005)





Instead of using the entire series, here we only use the data between January 1986 and December 2003 to illustrate causality testing using vector ARMA models. The series Y_t and X_t in the following discussion are defined as

Y_t : Non-seasonally adjusted monthly housing starts in U.S. between 1986 and 2003
X_t : Non-seasonally adjusted monthly houses sold in U.S. between 1986 and 2003.

During this sub-period, we have 216 observations for each series.

Following vector ARMA model identification procedures, the following seasonal vector ARMA model was obtained:

$$(\mathbf{I}-\phi B)\begin{bmatrix}\nabla\nabla_{12}Y_t\\ \nabla\nabla_{12}X_t\end{bmatrix}=(\mathbf{I}-\theta B)(\mathbf{I}-\Theta B^{12})\mathbf{a}_t\ . \qquad (7.9)$$

In the above model, the constant term is insignificant and hence omitted from the model. The model is first specified in the SCA System using the MTSMODEL command followed by the CAUSALTEST command

```
➔ MTSMODEL NAME VARMASEASONAL.  SERIES HSTART(1,12),HSOLD(1,12). @
     MODEL IS (1-PHI1*B)SERIES=(1-THETA1*B)(1-THETA2*B**12)NOISE.
➔ CAUSALTEST VARMASEASONAL.  OUTPUT PRINT(CORR).
```

```
 -----------------------
 ERROR COVARIANCE MATRIX
 -----------------------
               1           2
   1    26.917782
   2     7.226946   19.205436

 ==============================================================
 SUMMARY OF FINAL PARAMETER ESTIMATES AND THEIR STANDARD ERRORS
 ==============================================================
 PARAMETER             PARAMETER                   FINAL        ESTIMATED
  NUMBER              DESCRIPTION                ESTIMATE       STD.  ERROR
 ----------    -----------------------------     ------------    ------------
     1          AUTOREGRESSIVE ( 1, 1, 1)         -0.068799        0.103855
     2          AUTOREGRESSIVE ( 1, 1, 2)         -0.240153        0.200579
     3          AUTOREGRESSIVE ( 1, 2, 1)         -0.058576        0.068029
     4          AUTOREGRESSIVE ( 1, 2, 2)          0.417557        0.087700
     5          MOVING AVERAGE ( 1, 1, 1)          0.725354        0.094989
     6          MOVING AVERAGE ( 1, 1, 2)         -0.726185        0.194046
     7          MOVING AVERAGE ( 1, 2, 1)          0.018602        0.035440
     8          MOVING AVERAGE ( 1, 2, 2)          0.721436        0.076811
     9     SEAS MOVING AVERAGE (12, 1, 1)          0.927476        0.042646
    10     SEAS MOVING AVERAGE (12, 1, 2)         -0.070954        0.052423
    11     SEAS MOVING AVERAGE (12, 2, 1)          0.074544        0.038916
    12     SEAS MOVING AVERAGE (12, 2, 2)          0.812825        0.043798

CAUSALITY TEST BETWEEN VARIABLES  HSTART  AND  HSOLD

-2*(LOG LIKELIHOOD) UNDER H1,H2,H3,H3*,H4,H4*,H5,H5* ARE:
   0.17518177E+04
   0.17268096E+04
   0.17221825E+04
   0.17360510E+04
   0.16851187E+04
   0.17095935E+04
   0.16818872E+04
   0.16968655E+04

 RESULT BASED ON THE BACKWARD PROCEDURE ( Y:HSTART  ,  X: HSOLD   )
   HSTART <= HSOLD    (Y IS CAUSED BY X)

 RESULT BASED ON THE FORWARD PROCEDURE  ( Y:HSTART  ,  X: HSOLD   )
   HSTART <= HSOLD    (Y IS CAUSED BY X)
```

Based on the CAUSALTEST output, the values of $\ell(H_i)$ under various hypotheses are

$$\begin{aligned}
&\ell(H_1) = 1751.818\\
&\ell(H_2) = 1726.810\\
&\ell(H_3) = 1722.183\\
&\ell(H_3^*) = 1736.051\\
&\ell(H_4) = 1685.119\\
&\ell(H_4^*) = 1709.594\\
&\ell(H_5) = 1681.887\\
&\ell(H_5^*) = 1696.866\ .
\end{aligned} \tag{7.10}$$

Summarizing the SCA output of the log likelihood ratio statistics for various pairs of hypotheses and their corresponding χ^2 critical values at $\alpha = 5\%$, the following table is constructed:

$$\begin{aligned}
&LR(H_3 \text{ vs } H_5) = \ell(H_3) - \ell(H_5) = 40.295 > \chi^2_{0.05}(3) = 7.815 \quad \text{(significant)}\\
&LR(H_4 \text{ vs } H_5) = \ell(H_4) - \ell(H_5) = 3.231 < \chi^2_{0.05}(3) = 7.815 \quad \text{(insignificant)}\\
&LR(H_2 \text{ vs } H_3) = \ell(H_2) - \ell(H_3) = 4.627 < \chi^2_{0.05}(3) = 7.815 \quad \text{(insignificant)}\\
&LR(H_2 \text{ vs } H_4) = \ell(H_2) - \ell(H_4) = 41.691 > \chi^2_{0.05}(3) = 7.815 \quad \text{(significant)}\\
&LR(H_1 \text{ vs } H_2) = \ell(H_1) - \ell(H_2) = 25.008 > \chi^2_{0.05}(1) = 3.841 \quad \text{(significant)}\\
&LR(H_2 \text{ vs } H_5) = \ell(H_2) - \ell(H_5) = 44.922 > \chi^2_{0.05}(6) = 12.592 \quad \text{(significant)}\\
&LR(H_3^* \text{ vs } H_3) = \ell(H_3^*) - \ell(H_3) = 13.869 > \chi^2_{0.05}(1) = 3.841 \quad \text{(significant)}\\
&LR(H_4^* \text{ vs } H_4) = \ell(H_4^*) - \ell(H_4) = 24.475 > \chi^2_{0.05}(1) = 3.841 \quad \text{(significant)}\\
&LR(H_5^* \text{ vs } H_5) = \ell(H_5^*) - \ell(H_5) = 14.978 > \chi^2_{0.05}(1) = 3.841 \quad \text{(significant)}\\
&LR(H_1 \text{ vs } H_5) = \ell(H_1) - \ell(H_5) = 69.931 > \chi^2_{0.05}(7) = 14.067 \quad \text{(significant)}\ .
\end{aligned} \tag{7.11}$$

Using the above likelihood ratio statistics and both the backward and forward procedures suggested by Chen and Lee (1990), the CAUSALTEST concludes that housing starts is caused by houses sold.

CHAPTER 8

INTRODUCTION TO NONLINEAR TIME SERIES ANALYSIS IN THE SCA SYSTEM

Time series models, such as ARIMA and transfer function models discussed in Box and Jenkins (1976) and other literatures, typically assume the model parameters and innovation variances are constant (i.e., invariant) regardless of the time and the values of the series. While the behavior (processes) for a vast variety of real-life time series can be approximated by such constancy models, there are situations that different forms of time series models may be more useful in either understanding the behavior/relationships of the time series, or improving the accuracy of forecasting models. We collectively refer to this class of models as *nonlinear time series models*. It is useful to note that transfer function models in fact can capture certain types of nonlinear dynamic relationships between output and input time series. However it still assumes the parameters representing such nonlinear relationships remain the same (i.e., constancy parameters) regardless of the time and values of the series.

A classic issue in time series modeling is the non-homogeneity of innovation variances. When the innovation variances increase as the values of the time series increase, a class of power transformation (see e.g., Box and Jenkins 1976) is often find useful in both linearizing the relationships and homogenizing the variances. The issues related to power transformation are discussed in the next chapter (Chapter 9) as well as in Chapter 6. In some situations however (e.g., in financial time series analysis), our primary interest is to model and understand the behavior of innovation variances. Engle (1982), Bollerslev (1986), and others have developed a variety of conditional heteroscedastic models, generally known as ARCH/GARCH or their extended models. Such models are discussed in Chapter 10.

A common and more serious threat to the usability of traditional time series models is the non-constancy of the model parameters. The model parameters may evolve over time without a particular pattern. In some situations, however, the model parameters may remain constant (stable) during particular time frames, or seasons, or within certain ranges of input or output time series. Under such a situation, time-segmented or value-segmented time series models can be useful in the analysis and forecasting of such time series. Value-segmented time series models are commonly referred to as *threshold time series models*.

In Chapter 11, the TVPEXPLORE command is introduced to examine the potential time-varying (non-constancy) properties of the parameters in a time series model. Depending upon the information obtained in TVPEXPLORE analysis, we can then decide what sort of models could be employed for the next step of analysis. TVPEXPLORE not only provides a convenient method of studying the time-varying properties of estimated parameters, the information that is provides may help the analyst find hidden characteristics in the time series that can be useful in business analysis or essential to improve forecasting performance as well.

In Chapter 12, we discuss weighted estimation and forecasting method. This method is employed using the WESTIM and WFORECAST commands. Chapter 12 provides examples to show how WESTIM and WFORECAST commands are useful for time-segmented time series analysis and forecasting. The WESTIM command is very flexible and can be used for value-segmented modeling in various situations.

In Chapter 13, we discuss several special types of value-segmented time series models known as *threshold models*. In this chapter, examples of threshold autoregressive (TAR) models are shown. Other novel extensions are also discussed such as threshold transfer function models, and general threshold models with more complex noise models that go beyond the typical threshold autoregressive approach. The SCA commands presented in Chapter 13 include TARTEST, TARXTEST, THMTEST, THMEXPLORE, TARESTIM, and TARFORECAST commands.

CHAPTER 9

SEARCH FOR THE BEST POWER TRANSFORMATION FOR TIME SERIES FORECASTING

The Professional Edition (B) of the SCA Statistical System implements new capabilities to search for the best transformation that can be applied to a time series for the purpose of improving forecasting accuracy, linearizing (and thus simplifying) time series models, or improving estimation of intervention effects. In this chapter, the TSEARCH command is discussed. A related command, RETRANSFORM, to retransform forecasts back into original units was presented in an earlier chapter. A few supporting SCA analytic statements are also used to complete the examples. More information on SCA analytic statements is documented in the *SCA Reference Manual for Fundamental Capabilities*.

In univariate time series analysis, Box and Jenkins (1976) discussed the feasibility of obtaining an ARIMA model with homogenous residual variance based on a time series derived from a power transformed series similar to that proposed in Box and Cox (1964). Using Y_t to denote the original series with positive value (if Y_t is non-positive, a positive constant value can be added to Y_t before the transformation), this class of transformation can be expressed as

$$Z_t = \begin{cases} (Y_t^{\lambda} - 1)/\lambda & \text{if } \lambda \neq 0 \\ \ell n(Y_t) & \text{if } \lambda = 0 \end{cases} \tag{9.1}$$

where Z_t is the transformed series, λ (referred to as lambda) is the power value of the transformation, and t=1, 2, …, n. The above formulation of power transformations is continuous in the parameter λ. While the transformation in (9.1) has desirable properties such as continuity of λ at 0, the variances of transformed series are not comparable when λ's are different. To address this issue, the following scaled power transformation is recommended by Box and Jenkins (1976), and Ansley, Spivey, and Wrobleski (1977)

$$Z_t = \begin{cases} (Y_t^{\lambda} - 1)/(\lambda \dot{Y}^{\lambda-1}) & \text{if } \lambda \neq 0 \\ \dot{Y}\ell n(Y_t) & \text{if } \lambda = 0 \end{cases} \tag{9.2}$$

where $\dot{Y} = (Y_1\, Y_2\, \cdots\, Y_n)^{1/n}$ is the geometric mean of the series. We shall refer to the formulation of transformation in (9.1) and (9.2) as Type 1 power transformations.

In addition to using the above power transformation, the following formulation of power transformation is also considered

$$Z_t = \begin{cases} Y_t^{\lambda} & \text{if } \lambda \neq 0 \\ \ell n(Y_t) & \text{if } \lambda = 0 \, . \end{cases} \tag{9.3}$$

This form of power transformation has a discontinuity at $\lambda = 0$, and may possess some other undesirable properties. However this form of power transformation is simple, and therefore is frequently used. We shall refer to this formulation of transformation as Type 2 power transformations.

There are two primary issues in the application of the power transformations shown above. The first is to select an appropriate lambda value that will either improve the efficacy of the model or the accuracy of the forecasts. This issue is discussed in this chapter. The second issue is the correction of biases induced by the retransformation on the forecasts. More information on the RETRANSFORM command can be found in Chapter 6.

9.1 Procedures for Searching a Power Transformation

To search for an appropriate value of lambda for power transformation, Box and Jenkins (1976), and Ansley, Spivey, and Wrobleski (1977) suggest that the method of Box and Cox (1964) be used. This method is based on the maximization of a likelihood function (or equivalently minimization of residual standard error) which depends on lambda and the unknown parameters of the model. Details of the proof can be found in Ansley, Spivey, and Wrobleski (1977).

Assuming that a specific ARIMA model has already been chosen, Box and Jenkins (1976) show that the lambda value which minimizes the residual standard error of the scaled Z_t in (9.2) is the best power value of the transformation for the model considered. We shall also refer to such residual standard error as the within-sample RMSE (root mean squared error) which can be expressed as

$$\text{RMSE}(\lambda, \underline{\phi}, \underline{\theta}) = (\frac{1}{m} \sum (Z_t - \hat{Z}_t)^2)^{1/2} \tag{9.4}$$

where Z_t is the scaled transformed Y_t as shown in (6.2), $\hat{Z}_t$ is the one-step-ahead forecast of Z_t, and m is the effective number of observations used in computing the RMSE (thus m < n). The TSEARCH command of the SCA System provides the capability to compute the within-sample RMSE for a specific model over a range of lambda values. The lambda value that minimizes the RMSE's of the scaled Z_t is considered to be the best choice of the lambda value for the power transformation.

From a forecasting point of view, we may consider a criterion to select the lambda value based on the minimization of the RMSE of the original observations Y_t (Liu, 2006), which is

$$\text{RMSE}(\lambda, \underline{\phi}, \underline{\theta}) = (\frac{1}{m} \sum (Y_t - \hat{Y}_t)^2)^{1/2} \tag{9.5}$$

where $\hat{Y}_t$ is the retransformed one-step-ahead forecast $\hat{Z}_t$. The TSEARCH command also provides such a capability for the determination of the lambda value.

It is useful to note that the value of Y_t generally increases over time for most business and economic time series, therefore the forecast error $|Y_t - \hat{Y}_t|$ also increases over time. Consequently, the within-sample RMSE based on the original scale (i.e., using RMSE defined in (9.5)) is more influenced by the latter observations of the series and less influenced by the earlier observations. This implicitly imposes the *importance of time order* in searching the power transformation. This is not the case if the scaled Z_t is used in the minimization of RMSE since we expect ($Z_t - \hat{Z}_t$) to have a similar distribution throughout the entire transformed series. In forecasting, we are primarily interested in the accuracy of the most current forecasts. Hence, the forecasting accuracy (reflected by the one-step-ahead forecast errors) for the latter part of the series is more important than the earlier part. We expect the RMSE criterion in (9.5) to provide a better choice of lambda for forecasting applications compared to the lambda obtained by (9.4).

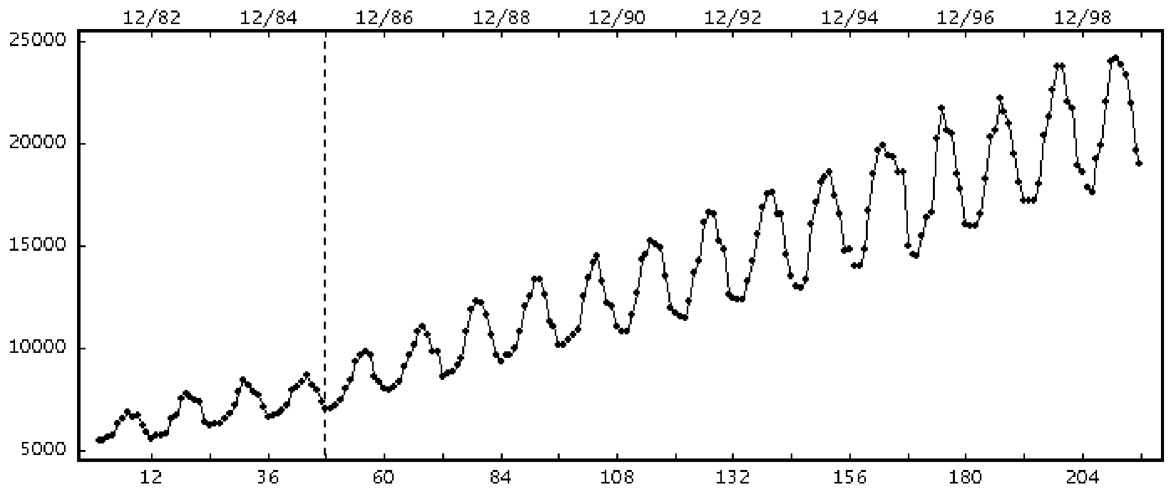
The power transformation in (9.1) and (9.2) are both Type 1 transformations and are basically the same (see Chapter 6 for the difference between Type 1 and Type 2 power transformation). The only difference between (9.1) and (9.2) is that the transformation in (9.2) is scaled by the geometric mean. Unlike Type 1 power transformations, Type 2 power transformations have a discontinuity at $\lambda = 0$, which renders it inappropriate to use when searching for a lambda value based on the minimization of the RMSE of the scaled Z_t. However, it is still appropriate to search for a lambda value based on the minimization of the RMSE of the original series Y_t. This is part of the rationale that leads us to strongly recommend a Type 1 transformation when considering power transformations in time series analysis and forecasting.

We shall use one example to illustrate the search of the lambda value for an appropriate power transformation. Additional examples can be found in Liu (2006).

9.2 Peak Electricity Load Forecasting Example of Power Transformation Search

In this section we provide an example of power transformation in time series analysis using the monthly electricity peak load data in a certain service area of Taiwan. The data analyzed are from January 1982 up to and including December 1999 (a total of 219 monthly observations). The original monthly peak load data (PEAKLOAD) is displayed below.

Time Series Plot of Monthly Electricity Peak Loads



By examining the above time series plot, we find the first four years of data have a different trend when compared with the rest of the time series. To simplify our discussion, first a power transformation analysis is conducted using the data starting in the fifth year which corresponds to time periods between t=49 and t=192. We reserve the data between t=193 and t=216 for post-sample forecasting comparison. Power transformation analysis that includes the first four years of data can be found in Liu (2006) and will be discussed later in this chapter as well. After performing a preliminary analysis, the following model is chosen for this time series which can be expressed as

$$(1-B)(1-B^{12})Y_t = (1-\theta_1 B)(1-\Theta_1 B^{12})a_t \; .$$

The model is specified in the SCA System using the following TSMODEL command

```
➔ TSMODEL MODEL1.  MODEL IS PEAKLOAD(1,12)=(1)(12)NOISE.
```

In the above command, MODEL1 is used to reference the specified model in the SCA System. The model reference is subsequently used in the TSEARCH command to find a power value (lambda) which is the best power transformation under this model.

Before the TSEARCH command is employed, an analytic statement is used in the SCA System to assign the value 1 to a variable named VTYPE. This variable will be used in the TSEARCH command shown below.

```
➔ VTYPE=1
➔ TSEARCH MODEL1.   POWER 0.05,-1.0,1.0.   FORM  VTYPE,GMEAN. @
     SPAN 49,192,216.   WITHIN-RMSE VPOWER,WRMSEO,WRMSET.   @
     POST-RMSE VPOWER, PRMSEO, PRMSET.
```

In the above TSEARCH command, we use the data between t=49 and t=192 (a total of 144 observations) to determine the lambda value. The last 24 observations (between t=193 and t=216) are reserved for computing the post-sample RMSE for forecasting. This is specified using the SPAN subcommand. The FORM subcommand is used to specify the type of transformation (TYPE=1) and whether the geometric mean should be used to scale the data. We indicate the use of geometric mean scaling by including a variable name (GMEAN is used here) as the second argument. After the TSEARCH command is executed, the GMEAN variable will hold the geometric mean value which can then be used later in an SCA session for retransformation or other applications.

The POWER subcommand is used to specify the range of power values (lambda values) that will be considered. The first argument specifies the increment of the lambda value and the second and third arguments specify the lower and upper bounds of the lambda values.

The WITHIN-RMSE and POST-RMSE subcommands are used to specify the variables to hold information related to the within-sample and post-sample RMSE's across the range of power values considered. The output results from the TSEARCH command are shown below. Some output has been suppressed for brevity.

```
THE FOLLOWING ANALYSIS IS BASED ON TIME SPAN    49  THRU   192
THE POST-SAMPLE  RMSE  IS BASED ON TIME SPAN   193  THRU   216

---------------------------------------------------
POWER TRANSFORMATION USING LAMDA (POWER):   -1.0000
---------------------------------------------------
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1      12
PEAKLOAD   RANDOM     ORIGINAL     (1-B  ) (1-B  )
-----------------------------------------------------------------------

PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1          PEAKLOAD   MA       1      1     NONE      .6470     .0665   9.73
  2          PEAKLOAD   MA       2     12     NONE      .5108     .0795   6.42

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           144
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           131
RESIDUAL STANDARD ERROR (WITHOUT OUTLIER ADJUSTMENT). .  0.486509E+03

LAMDA VALUE OF POWER TANSFORMATION=      -1.0000
WITHIN-SAMPLE RMSE (UNTRANSFORMED)= 0.651556E+03     N. OF OBS.=   131
POST-SAAMPLE  RMSE (UNTRANSFORMED)= 0.734450E+03     N. OF OBS.=    24
```

```
---------------------------------------------------
POWER TRANSFORMATION USING LAMDA (POWER):   -0.9500
---------------------------------------------------
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1      12
PEAKLOAD   RANDOM     ORIGINAL     (1-B  ) (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1          PEAKLOAD   MA       1      1     NONE      .6501     .0672   9.68
  2          PEAKLOAD   MA       2     12     NONE      .5228     .0780   6.70

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . . .           144
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . . .           131
RESIDUAL STANDARD ERROR (WITHOUT OUTLIER ADJUSTMENT). .  0.482516E+03

LAMDA VALUE OF POWER TANSFORMATION=      -0.9500
WITHIN-SAMPLE RMSE (UNTRANSFORMED)= 0.644508E+03     N. OF OBS.=   131
POST-SAAMPLE  RMSE (UNTRANSFORMED)= 0.725407E+03     N. OF OBS.=    24
     .
     .
     .
---------------------------------------------------
POWER TRANSFORMATION USING LAMDA (POWER):    0.9500
---------------------------------------------------
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1      12
PEAKLOAD   RANDOM     ORIGINAL     (1-B  ) (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1          PEAKLOAD   MA       1      1     NONE      .9865     .0097 101.50
  2          PEAKLOAD   MA       2     12     NONE      .5074     .0786   6.46

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . . .           144
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . . .           131
RESIDUAL STANDARD ERROR (WITHOUT OUTLIER ADJUSTMENT). .  0.559287E+03

LAMDA VALUE OF POWER TANSFORMATION=       0.9500
WITHIN-SAMPLE RMSE (UNTRANSFORMED)= 0.564448E+03     N. OF OBS.=   131
POST-SAAMPLE  RMSE (UNTRANSFORMED)= 0.620066E+03     N. OF OBS.=    24
```

```
  ---------------------------------------------------
  POWER TRANSFORMATION USING LAMDA (POWER):    1.0000
  ---------------------------------------------------
  SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
  -----------------------------------------------------------------------
  VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
            VARIABLE   OR CENTERED
                                          1      12
  PEAKLOAD   RANDOM     ORIGINAL     (1-B  ) (1-B  )
  -----------------------------------------------------------------------

   PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
     LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
    1          PEAKLOAD   MA       1      1     NONE      .9784      .0148  66.30
    2          PEAKLOAD   MA       2     12     NONE      .4975      .0793   6.27

  TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           144
  EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           131
  RESIDUAL STANDARD ERROR (WITHOUT OUTLIER ADJUSTMENT). .  0.569440E+03

  LAMDA VALUE OF POWER TANSFORMATION=       1.0000
  WITHIN-SAMPLE RMSE (UNTRANSFORMED)= 0.569440E+03     N. OF OBS.=   131
  POST-SAMPLE  RMSE (UNTRANSFORMED)= 0.620799E+03      N. OF OBS.=    24
```

Upon executing the above TSEARCH command, the within-sample RMSE based on the original series (WRMSEO), the within-sample RMSE based on the Box-Cox scaled transformation (WRMSET), and the post-sample RMSE based on the original data (PRMSEO) are stored in the SCA workspace. The PRINT command is used to display the results summary.

➔ PRINT POWER,WRMSEO,WRMSET, PRMSEO.

```
 VARIABLE     POWER    WRMSEO    WRMSET    PRMSEO
    ROW
     1       -1.000   651.556   486.509   734.450
     2        -.950   644.508   482.516   725.407
     3        -.900   636.692   478.575   721.012
     4        -.850   629.213   474.988   713.962
     5        -.800   621.536   471.678   707.540
     6        -.750   614.143   468.559   700.238
     7        -.700   606.763   465.690   693.165
     8        -.650   599.403   463.020   684.942
     9        -.600   592.168   460.623   676.430
    10        -.550   585.034   458.505   666.929
    11        -.500   578.049   456.673   656.588
    12        -.450   571.281   455.184   645.208
    13        -.400   564.832   454.070   632.860
    14        -.350   558.803   453.378   619.705
    15        -.300   553.332   453.161   606.019
    16        -.250   548.541   453.449   592.283
    17        -.200   544.411   454.277   579.160
    18        -.150   541.310   455.652   566.860
    19        -.100   539.055   457.570   556.329
    20        -.050   537.691   459.996   548.768
    21         .000   536.996   462.901   543.140
    22         .050   536.918   466.228   539.730
    23         .100   537.342   469.926   538.461
```

```
24        .150   538.157   473.938   539.115
25        .200   539.269   478.215   541.400
26        .250   540.602   482.710   544.951
27        .300   542.100   487.387   549.349
28        .350   543.714   492.215   554.173
29        .400   545.399   497.168   559.081
30        .450   547.113   502.223   563.840
31        .500   548.811   507.355   568.290
32        .550   550.447   512.535   572.297
33        .600   551.965   517.718   575.720
34        .650   553.288   522.837   578.394
35        .700   554.289   527.771   580.105
36        .750   554.666   532.236   579.899
37        .800   545.576   526.732   624.905
38        .850   552.012   537.507   619.311
39        .900   558.690   548.670   620.456
40        .950   564.448   559.287   620.066
41       1.000   569.440   569.440   620.799
```

Based on these results, we obtain a lambda value of 0.05 if the within-sample RMSE based on the original scale is used as the selection criterion. The corresponding post-sample RMSE is 539.730, which is very close to the smallest post-sample RMSE 538.461 (where $\lambda = 0.10$), and much smaller than the post-sample RMSE of 620.799 when no transformation is applied (i.e., when $\lambda = 1.00$). If the within-sample RMSE based on the scaled transformed data is used as a selection criterion for lambda, we obtain a lambda value of -0.30. The corresponding post-sample RMSE is 606.019, which is about 12% higher than the post-sample RMSE if $\lambda = 0.05$ is used. In the analysis shown above, we find that for forecasting applications, it is better to use the within-sample RMSE based on the original scale as a selection criterion for lambda rather than the traditional Box-Cox criterion which bases lambda selection using the scaled transformed series.

In some situations, the post-sample RMSE for a lambda that was selected based on the scaled transformed series may be higher than the post-sample RMSE of the non-transformed original series. This raises the question about the usefulness of traditional Box-Cox power transformation in forecasting applications. Using all data between t=1 and t=192 (i.e., not excluding the first four years of data), the within-sample RMSE based on the original series (WRMSEO), the within-sample RMSE based on the Box-Cox scaled transformation (WRMSET), and the post-sample RMSE based on the original data (PRMSEO) are computed by the TSEARCH command and summarized below.

```
 POWER    WRMSEO    WRMSET    PRMSEO
-1.000   568.140   400.624   770.280
  -.950   561.371   395.330   764.340
  -.900   555.078   390.197   760.422
  -.850   547.730   385.368   755.751
  -.800   540.831   381.113   751.246
  -.750   533.886   377.020   746.435
  -.700   526.602   373.269   740.699
  -.650   519.204   369.934   733.382
  -.600   511.615   367.009   724.031
  -.550   503.967   364.536   711.548
  -.500   496.428   362.638   695.359
```

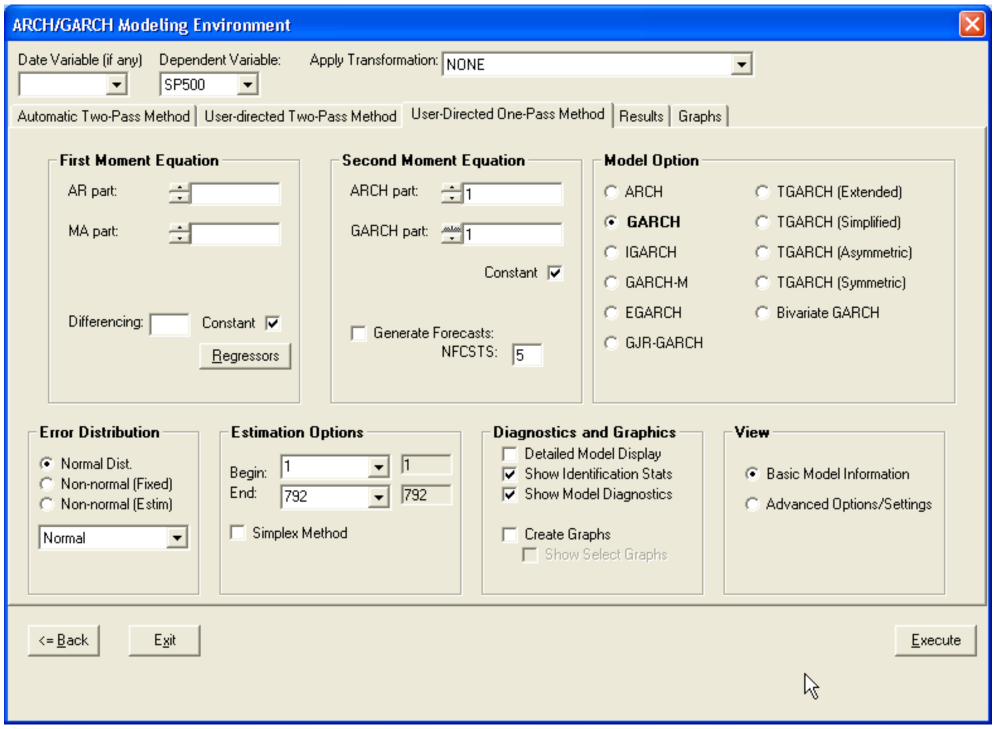
-.450	489.414	361.390	674.882
-.400	483.410	360.901	651.183
-.350	478.789	361.197	626.563
-.300	475.758	362.281	603.256
-.250	474.227	364.097	583.136
-.200	473.919	366.562	567.206
-.150	474.520	369.593	555.147
-.100	475.767	373.108	546.523
-.050	477.458	377.048	540.675
.000	479.445	381.366	536.990
.050	481.628	386.023	534.945
.100	483.939	390.997	534.136*
.150	486.333	396.270	534.258
.200	488.781	401.831	535.088
.250	491.261	407.671	536.460
.300	493.760	413.786	538.252
.350	496.268	420.172	540.373
.400	498.780	426.830	542.760
.450	501.291	433.758	545.363
.500	504.078	440.968	549.313
.550	506.485	448.433	551.698
.600	509.020	456.188	554.868
.650	511.489	464.219	557.878
.700	513.970	472.537	561.096
.750	516.434	481.143	564.353
.800	518.894	490.043	567.713
.850	521.347	499.242	571.139
.900	523.796	508.745	574.644
.950	526.242	518.557	578.220
1.000	528.685	528.685	581.872

From the above table, we obtain a lambda value of -0.20 if the within-sample RMSE based on the original scale is used as a selection criterion for lambda. The corresponding post-sample RMSE is 567.206. However, if the within-sample RMSE based on the scaled transformed series is used, we obtain a lambda value of -0.40. The corresponding post-sample RMSE is 651.183 which is much larger than the post-sample RMSE 581.872 if no transformation is applied to the time series (i.e., when $\lambda = 1.00$). Such a result gives practitioners reasons to question the usefulness of power transformations in forecasting. The lambda selected based on the original scale does not have this problem. More discussion of this issue can be found in Liu (2006).

CHAPTER 10

GARCH MODELING AND ANALYSIS

The Professional Edition (B) of the SCA Statistical System includes GARCH modeling, analysis, and forecasting capabilities through SCA WorkBench and the SCAB34S GARCH product. The SCAB34S product is a subset of capabilities from the B34S ProSeries Econometric System. More on the B34S family of products can be found in Stokes (1997). SCAB34S GARCH is included as an integrated component of the Professional Edition (B). The user is directed to the document, *General Autoregressive Conditional Heteroscedastic (GARCH) Modeling Using the SCAB34S-GARCH and SCA WorkBench*, for a detailed discussion of GARCH capabilities in the SCA System. A detailed and comprehensible discussion of GARCH models can be found in Liu (2006). An example of the ARCH/GARCH modeling environment is displayed below:


ARCH/GARCH Modeling Environment
Automatic Two-Pass Method
User-directed Two-Pass Method
User-Directed One-Pass Method
Results
Graphs
Regressors
<= Back
Exit
Execute

Date Variable (if any):
Dependent Variable: SP500
Apply Transformation: NONE

First Moment Equation
AR part:
MA part:
Differencing:
Constant ☑

Second Moment Equation
ARCH part: 1
GARCH part: 1
Constant ☑
☐ Generate Forecasts: NFCSTS: 5

Model Option
○ ARCH
● GARCH
○ IGARCH
○ GARCH-M
○ EGARCH
○ GJR-GARCH
○ TGARCH (Extended)
○ TGARCH (Simplified)
○ TGARCH (Asymmetric)
○ TGARCH (Symmetric)
○ Bivariate GARCH

Error Distribution
● Normal Dist.
○ Non-normal (Fixed)
○ Non-normal (Estim)
Normal

Estimation Options
Begin: 1 1
End: 792 792
☐ Simplex Method

Diagnostics and Graphics
☐ Detailed Model Display
☑ Show Identification Stats
☑ Show Model Diagnostics
☐ Create Graphs
☐ Show Select Graphs

View
● Basic Model Information
○ Advanced Options/Settings

The SCA System provides options to estimate a variety of conditional heteroscedastic models including the autoregressive conditional heteroscedastic (ARCH) model of Engle (1982), the generalized ARCH (GARCH) model of Bollerslev (1986), the integrated GARCH (IGARCH) model of Nelson (1990, 1991), the GARCH-M model of Engle, Lilien, and Robins (1987), the GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993), the exponential GARCH (EGARCH) model of Nelson (1991), and a variety of threshold GARCH models discussed in Zachoian (1994) and Tsay (2005). In addition, the non-normal error distributions including Student-t, Cauchy, and GED are supported.

CHAPTER 11

TIME SERIES MODELS WITH TIME-VARYING PARAMETERS

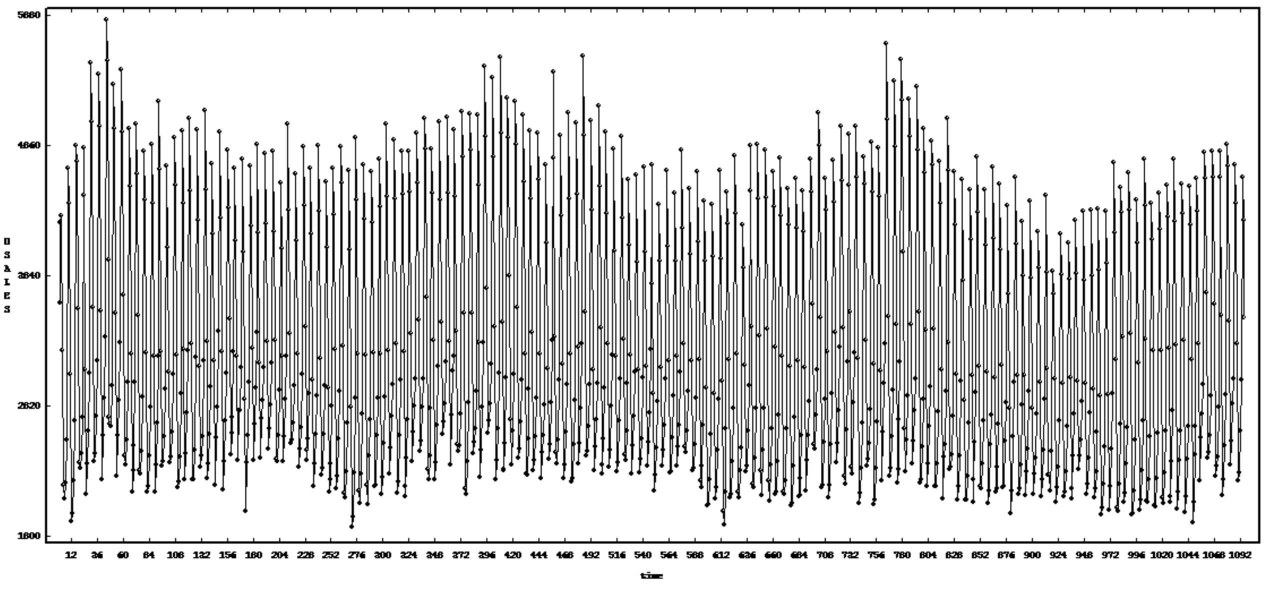
The Professional Edition (B) of the SCA Statistical System provides capabilities to employ time series models with time-varying parameters. In this chapter, the TVPEXPLORE command is presented. Other commands such as TSMODEL and ESTIMATE are also used to complete the illustrative examples. These additional commands are documented in the SCA reference manual, *Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 1*.

The TVPEXPLORE command is very useful to explore the stability and time-varying properties of the estimated model parameters. It can also be used to study seasonal effects, structural shifts, and overall model performance as it relates to both parameter estimates and windows of time.

11.1 Example of the TVPEXPLORE Command Using a Day-of-Week Effect Model

To illustrate an application of time-varying parameter analysis through the TVPEXPLORE command, we consider the daily per store sales (DSALES) of a service-related product from January 1, 2004 through December 31, 2006.

Daily Sales (1/1/2004 – 12/31/2006)



The plot reveals that DSALES has a very strong weekly periodic pattern and thus is dominated by strong day-of-week effects. Weekends (Friday to Sunday) have consistently higher sales than weekdays. In addition, Monday, Tuesday,…, and Sunday seem to have their own unique means, often referred to as *day-of-week effects*. In this example, we are interested in studying the characteristics of the day-of-week effects over time.

We begin by generating the day-of-week dummy variables using the DOWEEK and DVECTOR commands. The DATE and DSALES variables have been input into the SCA workspace prior to the commands shown below.

```
➔ DOWEEK DATE.  DAYOFWEEK VDOW.
➔ DVECTOR VDOW.  MAIN-EFFECT X.
```

The first twenty-one rows of the generated dummy variables (X1 to X6) are now printed along with the DATE, DSALES and VDOW variables for illustration. For more information on the use of the DOWEEK and DVECTOR commands, please refer to Chapter 2.

```
➔ PRINT VARIABLES DATE, DSALES,VDOW, X1 TO X6. @
   FORMAT '2F12.0,7F5.0'.  SPAN 1,21.
```

```
VARIABLE          DATE       DSALES  VDOW X1    X2    X3    X4    X5    X6
  ROW
    1        20040101        3632    4    0     0     0     1     0     0
    2        20040102        4261    5    0     0     0     0     1     0
    3        20040103        4314    6    0     0     0     0     0     1
    4        20040104        3257    7   -1    -1    -1    -1    -1    -1
    5        20040105        2203    1    1     0     0     0     0     0
    6        20040106        2093    2    0     1     0     0     0     0
    7        20040107        2220    3    0     0     1     0     0     0
    8        20040108        2555    4    0     0     0     1     0     0
    9        20040109        4690    5    0     0     0     0     1     0
   10        20040110        4413    6    0     0     0     0     0     1
   11        20040111        3070    7   -1    -1    -1    -1    -1    -1
   12        20040112        1922    1    1     0     0     0     0     0
   13        20040113        1981    2    0     1     0     0     0     0
   14        20040114        2236    3    0     0     1     0     0     0
   15        20040115        2712    4    0     0     0     1     0     0
   16        20040116        4869    5    0     0     0     0     1     0
   17        20040117        4740    6    0     0     0     0     0     1
   18        20040118        3591    7   -1    -1    -1    -1    -1    -1
   19        20040119        2379    1    1     0     0     0     0     0
   20        20040120        2417    2    0     1     0     0     0     0
   21        20040121        2505    3    0     0     1     0     0     0
```

We now specify a deterministic seasonal regression model with an AR(1) term using the TSMODEL command. The AR(1) term is used to accommodate day-to-day serial correlation. This model is then estimated using the ESTIMATE command.

```
➔ TSMODEL SALESMDL. MODEL IS @
     DSALES=C+(B1)X1+(B2)X2+(B3)X3+(B4)X4+(B5)X5+(B6)X6+1/(1;PH1)NOISE.
➔ ESTIMATE SALESMDL.
```

```
 SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  SALESMDL
 --------------------------------------------------------------------
  VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
            VARIABLE   OR CENTERED

   DSALES    RANDOM     ORIGINAL     NONE
     X1      RANDOM     ORIGINAL     NONE
     X2      RANDOM     ORIGINAL     NONE
     X3      RANDOM     ORIGINAL     NONE
     X4      RANDOM     ORIGINAL     NONE
     X5      RANDOM     ORIGINAL     NONE
     X6      RANDOM     ORIGINAL     NONE
  --------------------------------------------------------------------

   PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
     LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
    1    C               CNST      1      0     NONE  3212.6734   19.8714 161.67
    2    B1       X1     NUM.      1      0     NONE  -970.1566   10.2090 -95.03
    3    B2       X2     NUM.      1      0     NONE  -892.6729   10.2113 -87.42
    4    B3       X3     NUM.      1      0     NONE  -663.1560   10.2075 -64.97
    5    B4       X4     NUM.      1      0     NONE  -288.0881   10.2013 -28.24
    6    B5       X5     NUM.      1      0     NONE  1599.5879   10.1879 157.01
    7    B6       X6     NUM.      1      0     NONE  1152.0953   10.1881 113.08
    8   PH1     DSALES   D-AR      1      1     NONE      .7675     .0192  39.95

  EFFECTIVE NUMBER OF OBSERVATIONS . .          1095
  R-SQUARE . . . . . . . . . . . . . .         0.975
  RESIDUAL STANDARD ERROR. . . . . . .  0.152890E+03
```

The day-of-week effects ($\text{B1} \equiv \text{Monday}, \ldots, \text{B6} \equiv \text{Saturday}$) are estimated as deviations from the overall mean of DSALES (which is represented by C in the model). Since such a model uses the constraint B1+B2+…+B7=0, the Sunday effect is equal to –(B1+B2+…+B6). The use of the above deterministic seasonal model assumes that the day-of-week effects remain constant over time.

The TVPEXPLORE command can be used to explore the validity of the assumption of invariant day-of-week effects for the DSALES series. The WINDOWSIZE subcommand is used to specify the number of time periods in moving window estimation of the model parameters. The window size can be increased or decreased depending upon desired sensitivity in detecting the time-varying properties of the model parameters. In this example, the window size is specified as 84 observations (twelve weeks). Here, the first estimation will be from time span 1 to 84; the second estimation will be from 2 to 85; and so on. The parameter values and their t-values are saved for each estimation window in designated variables. All model parameters that have been specified with labels in the TSMODEL command will accumulate in this manner during the TVPEXPLORE estimation. Note that the first 83 values of the estimated parameters and their t-values are padded with missing values since the initial window size is specified as 84 observations.

```
➔ TVPEXPLORE SALESMDL.  WINDOWSIZE 84.  HOLD VARIANCE(VRNCE)
```

```
VARIABLE        C  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE       _C  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE       B1  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE      _B1  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE       B2  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE      _B2  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE       B3  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE      _B3  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE       B4  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE      _B4  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE       B5  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE      _B5  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE       B6  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE      _B6  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE      PH1  STORES PARAMETER ESTIMATES (WITH  1096 VALUES)
 VARIABLE     _PH1  STORES THE t-VALUES OF THE ABOVE ESTIMATES
 VARIABLE    VRNCE  STORES THE VARIANCE FOR EACH ESTIMATION WINDOW
```

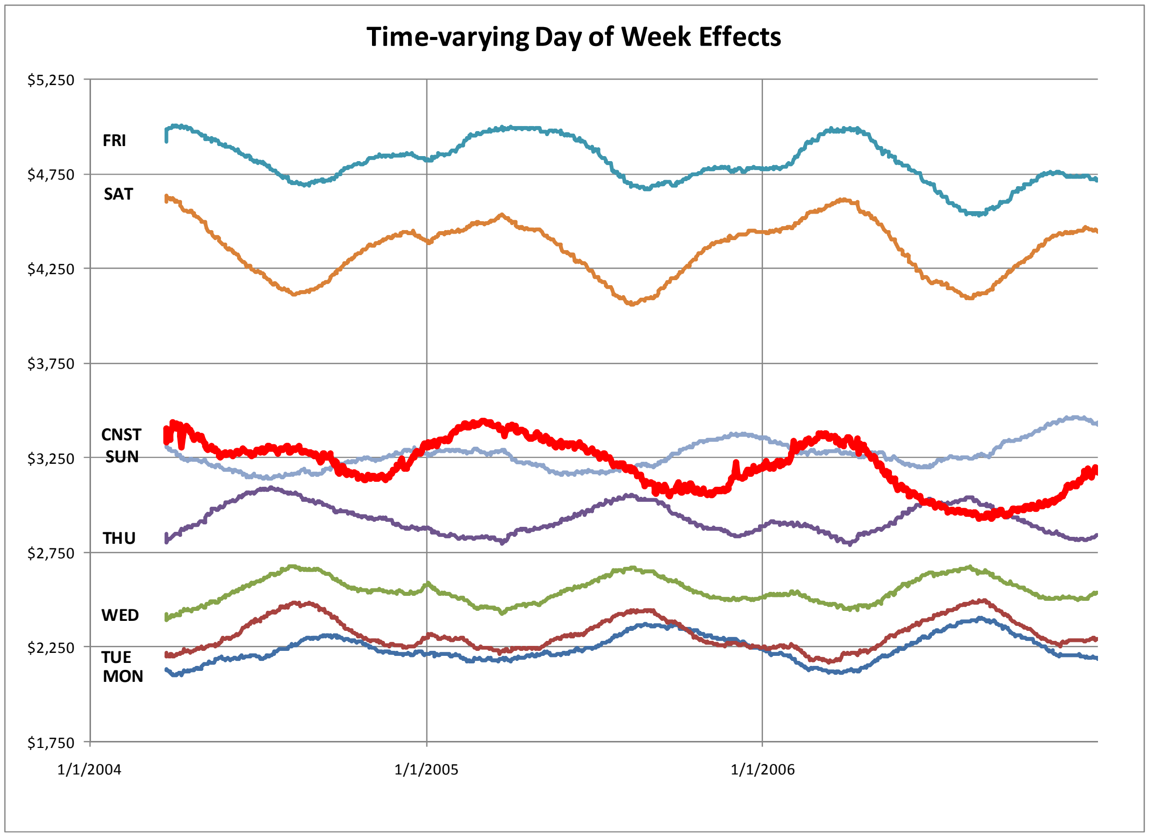
The above output echoes the names of the variables containing the parameter estimates, associated t-values, and residual variance. As mentioned earlier, the effect of Sunday (B7) can be computed using the following SCA analytic statement

```
➔ B7= -(B1+B2+B3+B4+B5+B6)
```

In order to better visualize the time-varying properties of the constant and day-of-week effects, the overall mean of the vector C is computed and added to the day-of-week effects using the following analytic statements

```
➔ CMEAN=MEAN(C)
➔ TB1=CMEAN+B1
➔ TB2=CMEAN+B2
➔ TB3=CMEAN+B3
➔ TB4=CMEAN+B4
➔ TB5=CMEAN+B5
➔ TB6=CMEAN+B6
➔ TB7=CMEAN+B7
```

The time-varying properties of the day-of-week effects can now be graphed using the SCAGRAF command or other graphing program.



The above plot provides an abundance of information regarding this daily sales series. First, we find that the day-of-week effects are not stable over time. Second, weekends (Friday through Sunday) have higher variability (and higher sales volumes) than weekdays. Third, the estimated constant (labeled as CNST in the graph) reveals that sales are decreased in 2006 in comparison with 2005.

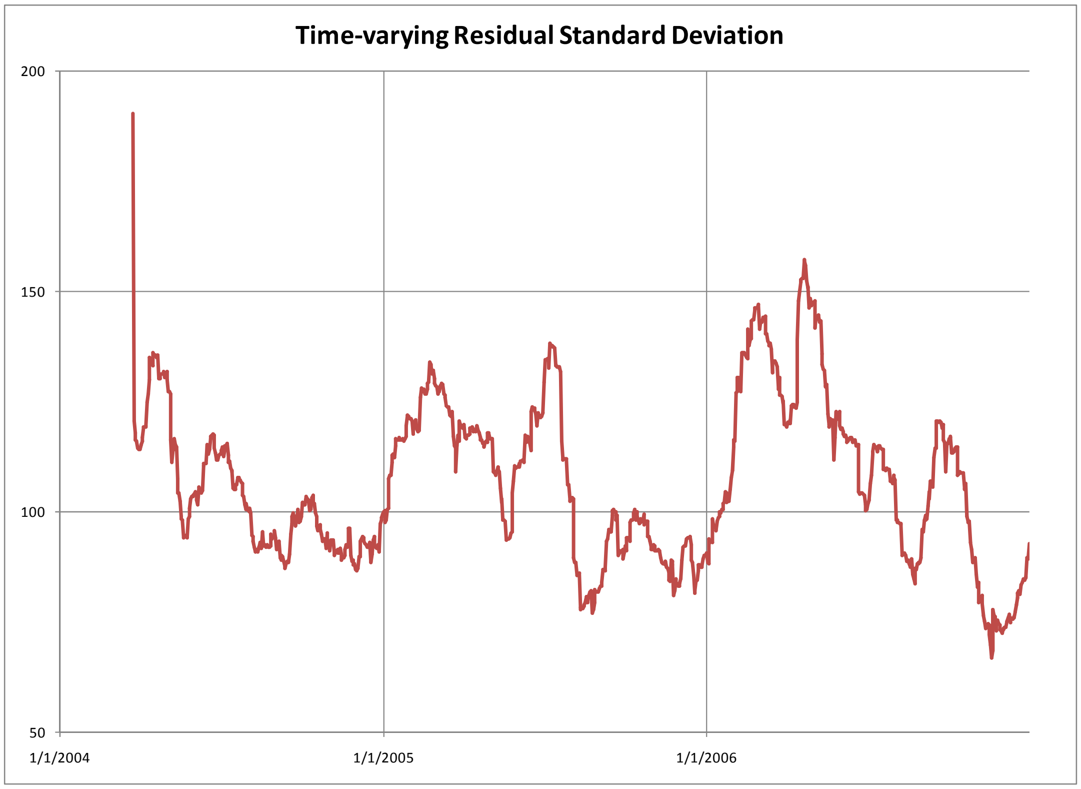
A yearly cyclical pattern is also present. Higher sales typically occur around February and lower sales typically occur around the September or October months. In addition, we find that the distribution of day-of-week effects across the week is somewhat dynamic. For example, during the peak cycle (around February), the relative increase of sales in the weekend periods are greater than the weekday periods. Also, during the yearly cycle with lower sales (around September or October), weekday sales are increasing at a relatively greater rate over weekend sales. Finally, it seems that Friday sales are eroding at a faster rate than other days and the gap between Friday and Saturday sales is somewhat narrowing.

As shown, the use of traditional deterministic models with TVPEXPLORE reveals much about the series at hand that may lead to better operational planning and business decisions. However, with such high instability among the day-of-week effects over time, it would be unwise to rely on such deterministic models for forecasting applications. Therefore, if forecasting is the primary application, adaptive models such as ARIMA are better suited. In addition, time-segmented or value-segmented (threshold) time series models may be useful in the analysis and forecasting of such series. A time-segmented model will be applied to the DSALES series to illustrate such application in the next chapter.

The residual variances for the estimation windows were also stored using the HOLD subcommand in the TVPEXPLORE command. Besides exploring the time-varying properties of the day-of-week effects (or other parameter estimates in the model), it may be useful to explore the residual standard errors of the model over time. In the TVPEXPLORE command, the residual variances were stored in the VRNCE variable. We now use the SQRT analytic statement to get the residual standard errors.

```
➔ STDERR=SQRT(VRNCE)
```

The residual standard errors (STDERR) can now be plotted using the SCAGRAF program or other graphing program as shown below.



The plot reveals that the residual standard errors are also not stable over time. The standard errors are relatively high earlier in each year but lower near the end of each year. This may be partially due to holidays and seasonal business conditions. With evidence of cyclical behavior in residual standard errors, we would also expect forecasting accuracy to be highly dependent upon the time of year in which it is measured. This information may be important to a business for seasonal planning, and to understand the underlying causes of such cyclical behavior in forecasting accuracy.

11.2 Example of TVPEXPLORE in the Context of Simple Moving Averages

The concept and power behind the TVPEXPLORE command is quite extraordinary. Even the use of a very simple model with the time-varying parameter approach can provide important information. Consider the following simple mean model

$$\mathrm{DSALES}_t = C + a_t. \qquad (11.1)$$

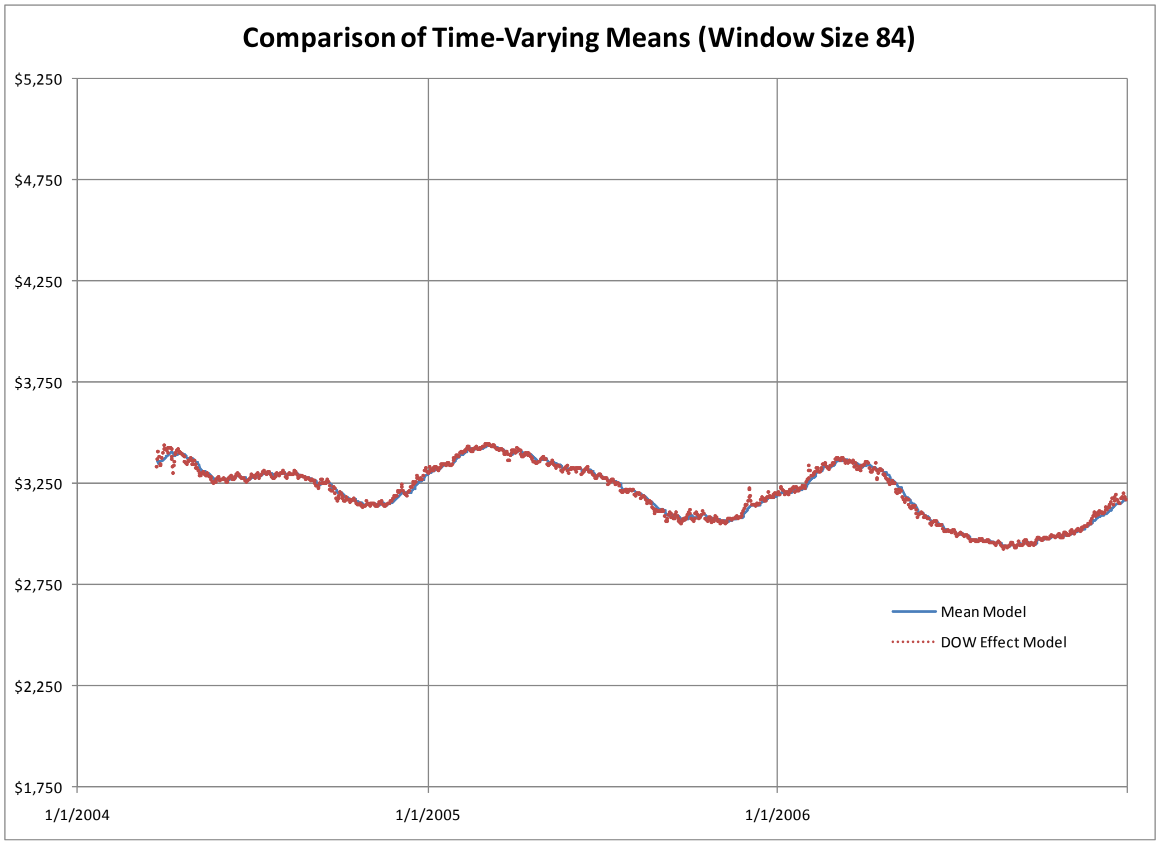
If the above model is applied to the DSALES series using the traditional estimation method, we obtain $\hat{C} = 3217.03$ as the estimate of the fixed mean. However, if we employ the TVPEXPLORE command with a moving window size of 84 observations, as in the previous example, we gain information on the year-to-year movement of the mean for the DSALES series similar to that shown in the previous model. Below are the SCA commands for such an analysis.

```
➔ TSMODEL MEANMDL. MODEL DSALES=C+NOISE.
➔ TVPEXPLORE MEANMDL. WINDOWSIZE 84.
```

In the table below, the first fourteen non-missing constant estimates from the model with day-of-week effects and the above simple mean model are displayed. Rows 1-83 are missing since the initial moving window size was specified as 84.

Row	Constant from Day-of-Week Model	Constant from Simple Mean Model	% Difference
84	3332.07	3361.89	-0.89%
85	3407.06	3352.19	1.61%
86	3382.19	3359.52	0.67%
87	3343.90	3360.71	-0.50%
88	3342.44	3360.17	-0.53%
89	3378.36	3359.57	0.56%
90	3399.23	3362.77	1.07%
91	3421.24	3368.36	1.55%
92	3438.69	3376.23	1.82%
93	3419.70	3382.39	1.09%
94	3411.95	3385.52	0.77%
95	3403.82	3387.66	0.47%
96	3429.59	3392.82	1.07%
97	3425.50	3397.65	0.81%

From the above table, we find the estimates for the mean are very similar in each window. To better illustrate the similarity between these two estimates, the mean estimates from the above two models using time-varying parameter estimation are displayed below.



From the above graphs, it is evident that the time-varying estimates of the mean are very similar to one another even though the models are of different form. The average difference of these two estimates is 0.9% in relative terms of percent. From the above graph, we can also see the systematic cyclical level movements from year to year. The estimates of time-varying means based on the simple mean model is the same as the traditional “moving average” approach. However, by including the time-varying day-of-week effects in the model (as shown in the first example), we gain more insight into the characteristics of the DSALES series without distorting the information provided by the simple mean model.

CHAPTER 12

WEIGHTED TIME SERIES MODEL ESTIMATION AND FORECASTING

The Professional Edition (B) of the SCA Statistical System includes weighted time series model estimation and weighted forecasting capabilities. The commands WESTIM and WFORECAST are documented in this chapter. These two commands can be employed for both time-segmented and value-segmented analysis and forecasting. Related commands including TARTEST, TARXTEST, TARESTIM, TARFORECAST, THMTEST, and THMEXPLORE are for value-segmented models discussed in the next Chapter. Additional information can be found in Liu (2006).

This chapter includes three examples to illustrate weighted time series model estimation and forecasting. The first example illustrates time-segmented analysis and forecasting of the daily sales series (DSALES), discussed in the previous chapter, by leveraging separate models for weekday and weekend periods. The second example uses stock market data to illustrate how the WESTIM command can be used to discount the effect of atypical sections of data in a time series without disrupting the serial correlations in the data. The third example illustrates how the WESTIM command can be used for a value-segmented (threshold) transfer function model to study the symmetrical (or asymmetrical) elasticity characteristics of new home sales with respect to interest rate increases or decreases.

Statistical modeling is used to capture homogeneous patterns or relationships that may exist in the data. However in time series data, such patterns may be dependent on the day of the week, or the month of the year, often referred to as *periodic time series* (Cleveland and Tiao, 1979). In other situations, such patterns or relationships may be temporarily disrupted by outliers or transient structural changes. If disruptions are isolated and not exceedingly large, outlier detection and adjustment techniques are sufficient to correct for the biases caused by such disruptions. However if the disruptions are clustered together or if their atypical effects are persistent over a period of time, it may be more appropriate to discount or disregard those portions of the data in time series modeling. In regression analysis, we can accomplish this by simply deleting such portions of the data during model estimation. In time series analysis, however, data cannot be arbitrarily deleted during model estimation due to the existence of serial correlation or seasonality. In this chapter, we introduce a weighted estimation method to facilitate the practice of discounting the effects of certain atypical data in ARIMA and transfer function modeling as well as in periodic time series data.

12.1 Model Estimation Using the Weighted Method

When the parameters in a time series model are estimated, the typical approach is to obtain the final parameter estimates that maximize the log likelihood function, or roughly speaking, minimize the sum of squared errors. For time series models, the sum of squared errors can be generally expressed as

$$\mathrm{SSE} = \sum_{t=t_1}^{n} \hat{a}_t^2 \tag{12.1}$$

where $\hat{a}_t$ is the residual (also referred to as estimated error) for the t-th observation, n is the total number of observations, and t_1 is the first residual in the model that can be computed. The above expression is not only appropriate for conditional least squares estimation, but also appropriate for exact maximum likelihood estimation because the entire log likelihood function can be cast in a form as that in (12.1).

In the weighted estimation method, instead of minimizing the sum of squared errors expressed in (12.1), we obtain the parameter estimates that minimize the following weighted sum of squared errors

$$\mathrm{SSE} = \sum_{t=t_1}^{n} w_t \hat{a}_t^2 \tag{12.2}$$

where $w_t = 1$ if the residual $\hat{a}_t$ is to be *included* in the computation of SSE, and $w_t = 0$ if the residual $\hat{a}_t$ is to be *excluded* in the computation of SSE. Even though an $\hat{a}_t$ is excluded in the computation of SSE, it is still used in computing the subsequent $\hat{a}_t$'s due to the serially correlated nature of time series. Obviously if $w_t = 1$ for all t, the weighted estimation method will produce the same estimates as the traditional method. By setting 1 or 0 for the values in the weight variable in (12.2), we can define the relevant segment of time series data to be studied, and obtain the parameter estimates that are optimal for this segment of time series data. We may view the weight variable w_t as an instrument for the convenient segmentation of a time series into certain homogenous groupings.

The weighted estimation method has many applications. For example, certain time series may possess a strong month-of-year effect, or day-of-week effect as shown in the previous chapter. By grouping those data that possess similar characteristics (e.g., weekdays and weekends), separate models can be entertained for those data groups which may help in better model estimates and in turn better forecasting performance. In the next section, we will discuss this more by revisiting the DSALES series used to illustrate the TVPEXPLORE command for time-varying parameters. Another use of the weighted estimation method may be to discount a portion of a time series in model estimation that exhibits a radically different pattern from the general series. We can do this by creating a weight variable consisting of zeros that correspond to observations whose effects on parameter estimation are to be discounted, and ones corresponding to observations that otherwise are to receive full weight during estimation.

The weight variable may also assume a more general structure rather than simply 0 and 1 values. A weight value may be varied between 0 and 1 to represent its relative importance. In addition, a weight variable may follow a step function or even a geometric sequence. By varying the structure of the weight variable, we can address a number of applications in time series modeling and forecasting.

12.2 Example of Time-varying Day-of-Week Effects in Daily Sales

In this section, we once again consider the daily sales series (DSALES). In Chapter 11, the TVPEXPLORE command was used to gain insight into the time-varying properties of the day-of-week effects of the DSALES series. It was found that weekend periods (Friday through Sunday) possessed different time-varying properties than weekday periods (Monday through Thursday). This leads us to consider whether the ARIMA model for the weekend periods is different from the ARIMA model for the weekday periods. Since sales during the weekend periods are very different from the weekday periods, a segmented model may result in improved forecasting accuracy for both weekdays and weekends. This example illustrates time-segmented models using the WESTIM command to obtain parameter estimates for weekday and weekend periods. The WFORECAST command is then used to combine the forecasts of these two models together.

We begin this illustration of the WESTIM command by creating binary weight variables to indicate weekday and weekend time periods for DSALES. The VDOW variable is a day-of-week indicator variable that was created earlier using the command DOWEEK (see Chapter 11). The following RECODE commands are used to generate the WEEKDAYS and WEEKENDS binary indicator variables.

```
➔ RECODE VDOW.  VALUES (1, 4, 1), (5, 7, 0).  NEW WEEKDAYS.
➔ RECODE VDOW.  VALUES (1, 4, 0), (5, 7, 1).  NEW WEEKENDS.
```

To better understand the binary variables created, the first twenty-one observations of the WEEKDAYS and WEEKENDS indicator variables are printed along with DATE, DSALES, and VDOW.

```
➔ PRINT DATE, DSALES, VDOW, WEEKDAYS, WEEKENDS.  SPAN 1, 21. @
     FORMAT '2F10.0, 3F9.0'
```

```
VARIABLE        DATE    DSALES     VDOW WEEKDAYS WEEKENDS
  ROW
    1      20040101      3632        4        1        0
    2      20040102      4261        5        0        1
    3      20040103      4314        6        0        1
    4      20040104      3257        7        0        1
    5      20040105      2203        1        1        0
    6      20040106      2093        2        1        0
    7      20040107      2220        3        1        0
    8      20040108      2555        4        1        0
    9      20040109      4690        5        0        1
   10      20040110      4413        6        0        1
   11      20040111      3070        7        0        1
   12      20040112      1922        1        1        0
   13      20040113      1981        2        1        0
   14      20040114      2236        3        1        0
   15      20040115      2712        4        1        0
   16      20040116      4869        5        0        1
   17      20040117      4740        6        0        1
```

```
    18       20040118       3591        7        0        1
    19       20040119       2379        1        1        0
    20       20040120       2338        2        1        0
    21       20040121       2450        3        1        0
```

Following appropriate model identification procedures, a seasonal model of the form

$$\left(1-B^7\right)\mathrm{DSALES}_t = \frac{\left(1-\theta_1 B\right)\left(1-\Theta_1^7\right)}{\left(1-\phi_1 B\right)}a_t \tag{12.3}$$

is found to be appropriate for the DSALES series. For comparison purposes, the above model is first estimated using all available data in a traditional manner without discerning between weekday and weekend periods. The model is specified in the SCA System using the TSMODEL command and is then estimated using the ESTIM command below.

➔ TSMODEL SALESMDL. MODEL DSALES(7)=(1)(7)/(1)NOISE.
➔ ESTIM SALESMDL. HOLD RESID(RES).

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  SALESMDL
--------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        7
 DSALES    RANDOM     ORIGINAL     (1-B  )
---------------------------------------------------------------------
 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1           DSALES    MA       1      1     NONE     -.4283     .0357 -11.99
  2           DSALES    MA       2      7     NONE      .5039     .0264  19.09
  3           DSALES   D-AR      1      1     NONE      .5025     .0319  15.77

EFFECTIVE NUMBER OF OBSERVATIONS . .          1088
R-SQUARE . . . . . . . . . . . . . .         0.984
RESIDUAL STANDARD ERROR. . . . . . .  0.120961E+03
```

The WESTIM command is now employed to estimate the parameters for the weekday model (Monday through Thursday). In order to maintain separate parameter estimates for the weekday and weekend models, the model in (12.3) is re-specified using the model name WKDAYMDL. The WEEKDAYS indicator variable is specified in the WEIGHT subcommand to obtain weighted parameter estimates for the weekday periods.

➔ WESTIM WKDAYMDL. WEIGHT WEEKDAYS.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  WKDAYMDL
---------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        7
 DSALES    RANDOM     ORIGINAL     (1-B  )
---------------------------------------------------------------------
```

```
 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT              ERROR  VALUE
  1           DSALES    MA       1      1     NONE     -.3639     .0424  -8.58
  2           DSALES    MA       2      7     NONE      .7090     .0308  23.03
  3           DSALES   D-AR      1      1     NONE      .6256     .0368  16.99

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .          1096
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .          1088
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.127417E+03
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.995442E+02
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           620
```

The WKENDMDL is now specified in similar manner. However, the WEEKENDS indicator variable is specified in the WEIGHT subcommand to obtain parameter estimates for the weekend periods.

```
➔ TSMODEL WKENDMDL.  MODEL DSALES(7)=(1)(7)/(1)NOISE.
➔ WESTIM WKENDMDL.  WEIGHT WEEKENDS.
```

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  WKENDMDL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        7
 DSALES    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT              ERROR  VALUE
  1           DSALES    MA       1      1     NONE     -.5145     .0568  -9.06
  2           DSALES    MA       2      7     NONE      .4366     .0425  10.26
  3           DSALES   D-AR      1      1     NONE      .3974     .0520   7.65

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .          1096
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .          1088
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.121978E+03
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.140483E+03
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           468
```

The model information from the three ARIMA model estimations are summarized below in the following table.

Model	$\hat{\phi}_1$	$\hat{\theta}_1$	$\hat{\Theta}_1$	RSE
All Days	.5025	-.4283	.5039	120.961
Weekdays	.6256	-.3639	.7090	99.544
Weekends	.3974	-.5145	.4366	140.483

As shown in the above summary table, the parameter estimates are quite different between the weekday and weekend models. In particular, the seasonal moving-average parameter estimate, $\hat{\Theta}_1$, for the weekday model is much larger than the seasonal parameter estimate for the weekend model. The t-value of $\hat{\Theta}_1$ for the weekday model is also much larger. This information reveals that the week-to-week changes in sales are much more stable for the weekday periods than the weekend periods. Also, the residual standard error (RSE) of the weekday model is clearly smaller than the RSE of the weekend model. Such information is otherwise masked by the averaging effect of the traditional (All Days) model. In operations planning or inventory management applications, the forecasting interval is often as important as the point forecast in determining customer service level and safety stock. The traditional model would result in a larger forecasting interval for weekday periods and a smaller forecasting interval for weekend periods than necessary.

Based on the above results, we may be inclined to leverage models based on weighted estimation to improve forecasting accuracy. The SCA System provides the WFORECAST command to combine forecasts from more than one model using a weighting method. In the WFORECAST command provided below, the WKDAYMDL and WKENDMDL models are forecasted. Prior to executing the WFORECAST command, the forecast dates are generated using the DATEBUILD command and the corresponding day of week information is stored in the variable DAYCODE. The RECODE command can then be used to generate the binary indicator variables for the weekday and weekend models.

```
➔ DATEBUILD FCSTDATE.  BEGIN 2007, 1, 1.  NOBS 14.   HOLD DOWEEK(DAYCODE).
➔ RECODE DAYCODE.  VALUES (1,4,1),(5,7,0).   NEW FIWKDAY.
➔ RECODE DAYCODE.  VALUES (1,4,0),(5,7,1).   NEW FIWKEND.
```

The weight variables (FIWKDAY and FIWKEND) specified in the WEIGHTS subcommand in the WFORECAST command below must correspond with the order of the regime models entered. Upon executing the WFORECAST command, interim forecasts are generated for the weekday model and the weekend model. Lastly, the interim forecasts are combined into the final forecasts based on the weight variables. Since the weight variables are of binary form with no overlap between weekday and weekend periods, the final forecasts are a direct copy of the appropriate interim forecasts.

```
➔ WFORECAST WKDAYMDL, WKENDMDL.  @
    WEIGHTS FIWKDAY, FIWKEND.  NOFS 14.
```

```
-------------------------------------------------
MODEL WKDAYMDL:    14 FORECASTS, BEGINNING AT 1096
-------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
 1097   2374.6762     99.5442
 1098   2436.5593    140.0418
 1099   2662.9423    153.0013
 1100   2965.9725    157.7841
 1101   4738.8807    159.6171
 1102   4461.5719    160.3289
 1103   3467.6502    160.6066
 1104   2281.2279    164.3494
 1105   2378.0958    167.5050
 1106   2626.3660    168.7240
 1107   2943.0894    169.1988
 1108   4724.5645    169.3842
 1109   4452.6154    169.4567
 1110   3462.0468    169.4851
---------------------------------------------------
MODEL WKENDMDL:    14 FORECASTS, BEGINNING AT 1096
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
 1097   2435.2684    140.4826
 1098   2429.8538    190.1211
 1099   2685.5587    196.8193
 1100   3011.6903    197.8564
 1101   4689.1343    198.0197
 1102   4387.7981    198.0455
 1103   3466.3836    198.0496
 1104   2360.6012    213.4665
 1105   2400.1805    225.4014
 1106   2673.7663    227.2291
 1107   3007.0039    227.5164
 1108   4687.2719    227.5617
 1109   4387.0580    227.5689
 1110   3466.0894    227.5700
---------------------------------------------------
COMBINED FORECASTS:   14 FORECASTS, BEGINNING AT 1096
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
 1097   2374.6763     99.5442
 1098   2436.5593    140.0418
 1099   2662.9424    153.0013
 1100   2965.9724    157.7841
 1101   4689.1343    198.0197
 1102   4387.7983    198.0455
 1103   3466.3835    198.0496
 1104   2281.2280    164.3494
 1105   2378.0957    167.5050
 1106   2626.3660    168.7240
 1107   2943.0894    169.1988
 1108   4687.2720    227.5617
 1109   4387.0581    227.5689
 1110   3466.0894    227.5700
```

It is also possible to construct binary indicator variables with non-zero weights that overlap one or more particular forecast periods. By doing so, the final forecast for those particular periods would be computed from the proportional contribution of the weights using the WFORECAST command. For example, it is possible to smooth the transition of forecasts from Thursday to Friday by averaging the Thursday interim forecasts of both the weekday and weekend models. To accomplish this, a new weight variable (FIWKEND) is generated that sets FIWKEND=1 for Thursday through Sunday periods, 0 otherwise. With the FIWKDAY and FIWKEND weight variables overlapping on Thursday, the proportional contribution of forecasts for Thursday periods is 1/(1+1) or 50%. Therefore, the final forecast would be the average of interim forecasts for the weekday and weekend regime models for all Thursday forecasts. Below, the new FIWKEND weight variable is used in the WFORECAST command. Note, that we are still using the regime models that were estimated based on the original WEEKDAYS and WEEKENDS binary indicator variables.

➔ WFORECAST WKDAYMDL,WKENDMDL. WEIGHTS FIWKDAY, FIWKEND. NOFS 14.

```
---------------------------------------------------
MODEL WKDAYMDL:    14 FORECASTS, BEGINNING AT 1096
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
 1097   2374.6762     99.5442
 1098   2436.5593    140.0418
 1099   2662.9423    153.0013
 1100   2965.9725    157.7841
 1101   4738.8807    159.6171
 1102   4461.5719    160.3289
 1103   3467.6502    160.6066
 1104   2281.2279    164.3494
 1105   2378.0958    167.5050
 1106   2626.3660    168.7240
 1107   2943.0894    169.1988
 1108   4724.5645    169.3842
 1109   4452.6154    169.4567
 1110   3462.0468    169.4851
---------------------------------------------------
MODEL WKENDMDL:    14 FORECASTS, BEGINNING AT 1096
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
 1097   2435.2684    140.4826
 1098   2429.8538    190.1211
 1099   2685.5587    196.8193
 1100   3011.6903    197.8564
 1101   4689.1343    198.0197
 1102   4387.7981    198.0455
 1103   3466.3836    198.0496
 1104   2360.6012    213.4665
 1105   2400.1805    225.4014
 1106   2673.7663    227.2291
 1107   3007.0039    227.5164
 1108   4687.2719    227.5617
 1109   4387.0580    227.5689
 1110   3466.0894    227.5700
---------------------------------------------------
COMBINED FORECASTS:   14 FORECASTS, BEGINNING AT 1096
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
 1097   2374.6763     99.5442
 1098   2436.5593    140.0418
 1099   2662.9424    153.0013
 1100   2988.8313    178.9455   —   Forecast 2988.8313 = (2965.9725*0.50)+(3011.6903*0.50)
 1101   4689.1343    198.0197
 1102   4387.7983    198.0455
 1103   3466.3835    198.0496
 1104   2281.2280    164.3494
 1105   2378.0957    167.5050
 1106   2626.3660    168.7240
 1107   2975.0466    200.4893   —   Forecast 2975.0466 = (2943.0894*0.50)+(3007.0039*0.50)
 1108   4687.2720    227.5617
```

```
  1109   4387.0581    227.5689
  1110   3466.0894    227.5700
```

It may also be useful to combine the Monday forecasts from the weekday model and weekend model to smooth this forecast between regimes. This can be achieved by simple modification of the weight indicator variables for the forecasts similar to above.

So far we used the same model form for both weekday and weekend models. However, the user is not limited to the same model for each regime. In this case, an alternative weekend model could be entertained for the DSALES series to improve forecasting further. The following alternative model (WKENDMDL) for the weekend periods could be specified and used in weighted estimation and forecasting, for instance.

➔ TSMODEL WKENDMDL. MODEL DSALES(7)=1/(1,2,3)(7)NOISE.
➔ WESTIM WKENDMDL. WEIGHT WEEKENDS.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- WEEKEND
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        7
 DSALES    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
   1          DSALES   D-AR      1      1      NONE      .9679     .0485  19.94
   2          DSALES   D-AR      1      2      NONE     -.3390     .0700  -4.84
   3          DSALES   D-AR      1      3      NONE      .1127     .0557   2.02
   4          DSALES   D-AR      2      7      NONE     -.4767     .0414 -11.53

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .          1096
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .          1079
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.116178E+03
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.128847E+03
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           463
```

The residual standard errors (RSE) from the above model estimations are summarized below to provide an indication of the improvement gained in modeling the DSALES series using weighted estimation.

Model	RSE	Cases	Cases%	WRSE
All Days	120.961	1088	100%	120.961
Weekdays	99.544	620	57%	56.74
Weekends(1)	140.483	466	43%	60.41
Weekends(2)	128.847	466	43%	55.40

From the above summary table, weighted residual standard errors (WRSE) can be computed based on the effective number of cases in each regime. Since the effective number of cases is slightly different between the two weekend models, the average number of cases (466) is used for comparison purposes. By adding the WRSE values of the weekday model and weekend model, we can compare the relative improvement between the "All Days" model and the segmented models. Depending on the model used for the weekend periods, the weighted estimation provides a relative improvement of 3.2% ((56.74+60.41)/120.961) to 9.3% ((56.74+55.40)/120.961) in residual standard error over the traditional "All Days" model. Further improvements are also possible by refining the weekday model as we have done for the weekend model.

If each day of the week follows its own distinct pattern, an individual model can be employed for each day (Monday, Tuesday,…,Sunday). Such models are known as *periodic time series models* and have been discussed extensively by Cleveland and Tiao (1979), Tiao and Grupe (1980), and others. In this example, the weekday and weekend periods were grouped because of their similar properties as seen in the day-of-week effects plot presented in the previous chapter.

12.3 Example of Weighted Estimation with Outlier Adjustment Using Stock Market Data

In this section we illustrate the use of weighted estimation in conjunction with joint outlier adjustment. The data used in this example consist of monthly stock market series from January 1976 up to and including December 1991 (a total of 192 observations for each series). The three time series are:

(1) The monthly average of the Standard and Poor's 500 stock index,

(2) The monthly average of long-term government security interest rates (from the Federal Reserve Bulletin), and

(3) The monthly composite index of leading indicators (from Business Conditions Digest).

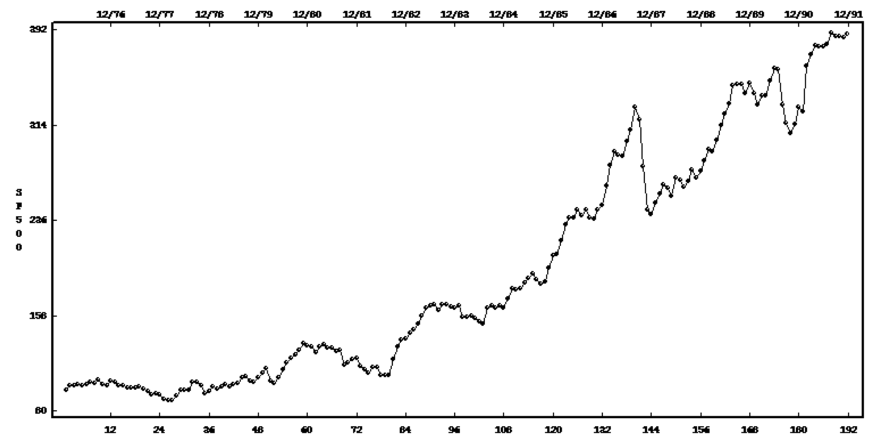
In this data set, we are interested in understanding how monthly stock prices (reflected by S&P500 index) are influenced by long-term interest rates and general economic conditions (reflected by the monthly composite index of leading indicators).

The data are displayed below and stored in the SCA workspace under the labels SP500, IRLONG and LINDCTR, respectively. From the time plots, we see that SP500 increases steadily until observation 142, at which time it plummets for three consecutive periods. This period corresponds to the stock market crash in October-December 1987. Since these observations behave abruptly, special consideration must be rendered to such data points during model estimation. In the previous example, we were concerned to best accommodate weekday and weekend patterns in the data by developing two regime models. In this example, we are interested in discounting a section of the data because it disrupts the otherwise stable relationship in the data.

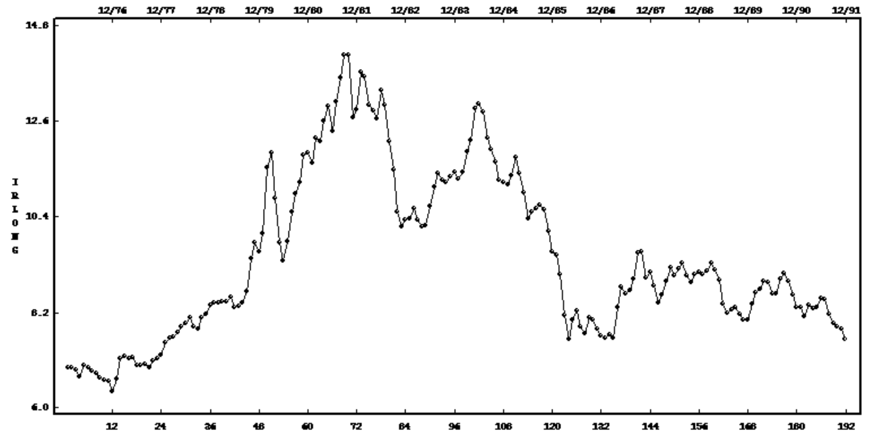
We will analyze the natural logarithms of all time series. In this way, we can assess the percent change in the response for a 1% change in an explanatory variable. The log transformed series of SP500, IRLONG and LINDCTR are stored in the SCA workspace under the labels LNSP500, LNIRLONG and LNLINDTR respectively.

Time Series Plots of Stock Market Data

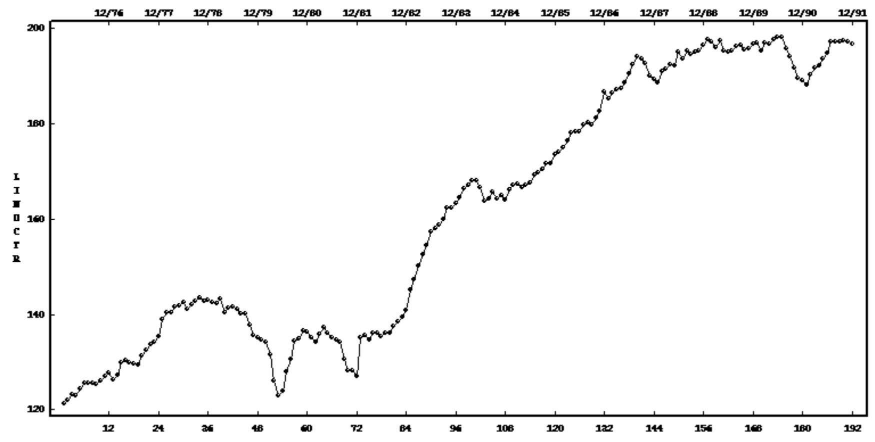
SP500



IRLONG



LINDCTR



Using the LTF method (Liu et. al. 1992) and the data during the first 141 months, we identify the following transfer function model to be appropriate in representing the relationship between S&P500, long-term interest rates and composite leading indicators.

$$(1-B)\mathrm{LNSP500}_t = C + \beta_1(1-B)\mathrm{LNIRLONG}_t + \beta_2(1-B)\mathrm{LNINDTR}_t + (1-\theta B)a_t \tag{12.4}$$

The estimation results for the above model indicate that both long-term interest rates and leading indicators influence stock prices significantly. However in reviewing the time series plots for SP500, IRLONG, and LINDTR, we find that during the period 1980 to 1982 the long-term interest rates are quite high which coincides with the low leading indicator periods. It would be interesting to evaluate the estimation results if this part of the data is disregarded from the analysis. Furthermore we may also wish to lessen the impact of some data points related to the 1987 stock market crash in order to minimize any atypical effects of such a major event. The following GENERATE command is used to create a weight variable (WEIGHT1) that consists of 1's, except for those time periods between t=49 and t=85 (stagnated economy period of 1980-1982), and t=142 and t=147 (stock market crash of 1987). The weights for these time periods are 0's. This weight variable allows us to estimate the parameters with zero weights assigned to those atypical observations.

```
➔ GENERATE  VARIABLE IS WEIGHT1.  NROW IS 192.  @
      VALUES ARE 1 FOR 48, 0 FOR 37, 1 FOR 56, 0 FOR 6, 1 FOR 45.
```

The model in (12.4) can be specified and estimated using the following TSMODEL and WESTIM commands. In the WESTIM command, the WEIGHT subcommand is used to specify the weight variable (WEIGHT1) that will be employed during model estimation. The OADJUST subcommand is specified with the keyword “JOINT” to employ the joint estimation method of outlier effects. The user also has the option to specify the keyword “SEQUENTIAL” to employ the sequential estimation method of outlier effects, or the keyword “NONE” to not employ outlier detection and estimation.

```
➔ TSMODEL  STOCKMDL.  NO SHOW.  @
      MODEL LNSP500(1)=C+(0)LNIRLONG(1)+(0)LNLINDTR(1)+(1)NOISE.
➔ WESTIM  STOCKMDL.  WEIGHT IS WEIGHT1.  OADJUST JOINT.
```

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- STOCKMDL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1
LNSP500    RANDOM     ORIGINAL     (1-B  )
                                        1
 LNLONG    RANDOM     ORIGINAL     (1-B  )
                                        1
 LNLEAD    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

  PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD      T
   LABEL        NAME    DENOM.                TRAINT               ERROR  VALUE
  1    C                CNST      1      0     NONE      .0069      .0025   2.79
  2         LNIRLONG   NUM.      1      0     NONE     -.4752     .0704  -6.75
  3         LNLINDTR   NUM.      1      0     NONE      .4743     .2538   1.87
  4          LNSP500    MA       1      1     NONE     -.2353     .0807  -2.92

SUMMARY OF OUTLIER DETECTION AND ADJUSTMENT
-------------------------------------
 TIME    ESTIMATE   T-VALUE    TYPE
-------------------------------------
   50      0.081      5.38      AO
   69     -0.078     -3.29      LS
   81      0.079      3.22      IO
  142     -0.127     -5.19      IO
  143     -0.136     -5.55      IO
  182      0.087      3.53      IO
-------------------------------------

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           192
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           191
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.310057E-01
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.232833E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           148
```

If we wish to estimate model (12.4) with outlier adjustment but without omitting any data, then the OESTIM command can be used. In this instance, the following results are obtained.

➔ OESTIM STOCKMDL.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- STOCKMDL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1
LNSP500    RANDOM     ORIGINAL     (1-B  )
                                        1
LNIRLONG   RANDOM     ORIGINAL     (1-B  )
                                        1
LNLINDTR   RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------
PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1    C              CNST      1      0     NONE      .0066     .0021   3.13
  2          LNIRLONG  NUM.      1      0     NONE     -.3705     .0506  -7.32
  3          LNLINDTR  NUM.      1      0     NONE      .9559     .1660   5.76
  4          LNSP500    MA       1      1     NONE     -.2437     .0731  -3.33

SUMMARY OF OUTLIER DETECTION AND ADJUSTMENT
-------------------------------------
 TIME    ESTIMATE   T-VALUE    TYPE
   50      0.075      5.30      AO
   69     -0.047     -3.31      AO
   73     -0.100     -4.34      IO
   81      0.079      3.42      IO
   98     -0.071     -3.16      LS
  142     -0.124     -5.39      IO
  143     -0.122     -5.29      IO
  182      0.084      3.63      IO
```

```
TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           192
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           191
RESIDUAL STANDARD ERROR (WITHOUT OUTLIER ADJUSTMENT). .  0.304104E-01
RESIDUAL STANDARD ERROR (WITH OUTLIER ADJUSTMENT) . . .  0.230039E-01
```

The results from the two commands above reveal several interesting findings. Both estimation results show that long-term interest rates have a strong influence on stock market prices. The former (with weighted estimation) shows that a 1% relative increase in long-term interest rates may decrease the stock prices by 0.4752%, and the latter (without a weight variable) shows that the elasticity due to long-term interest rates is only 0.3705%. More importantly, when weighted estimation is considered (i.e., certain portions of the data have their effects discounted), the leading indicators do not show a significant influence on stock market prices, while the estimation without discounting any data indicates a strong positive relationship. The differences between these results are somewhat inconsistent but reveal quite interesting information. If we examine the time series plots of these three variables carefully, we find the data disregarded during 1980-82 happen to correspond to a low or downward period of composite leading indicators. The rest of the composite leading indicators are generally in an upward trend. When data during 1980-1982 (and stock market crash period) are discounted, the leading indicators become insignificant. This implies that stock prices are not affected significantly by composite leading indicators in typically good economic conditions, but are sensitive to composite leading indicators in severely bad economic conditions (during 1980-82).

It would also be interesting to investigate the results if the WESTIM command without outlier adjustment is used for the model in (12.4) as shown below.

➔ WESTIM STOCKMDL. WEIGHT IS WEIGHT1.

The parameter estimates are summarized below. We find they are rather close to those obtained with outlier adjustment using the WESTIM command. The estimate $\hat{\beta}_2$ is only marginally significant. Thus weighted estimation alone alleviates some biases caused by outliers in this example.

$$\begin{aligned}
\hat{C} &= 0.0073 && (t = 2.87) \\
\hat{\beta}_1 &= -0.4926 && (t = -6.68) \\
\hat{\beta}_2 &= 0.5775 && (t = 2.17) \\
\hat{\theta} &= -0.2257 && (t = -2.77) \\
\hat{\sigma} &= 0.02318
\end{aligned}$$

If we wish to estimate model (12.4) without using the weighted method and without outlier adjustment, then the following standard ESTIM command can be specified

➔ ESTIM STOCKMDL.

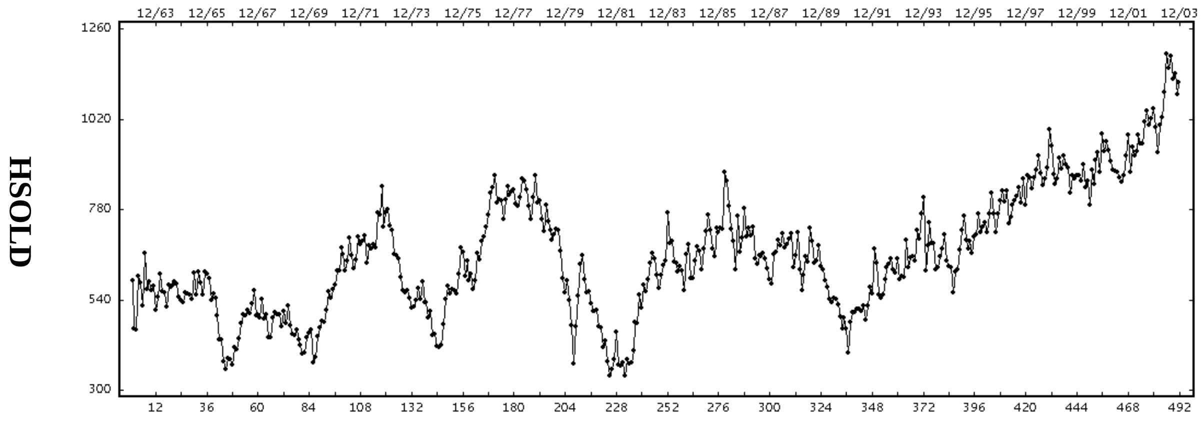
The parameter estimates are summarized below:

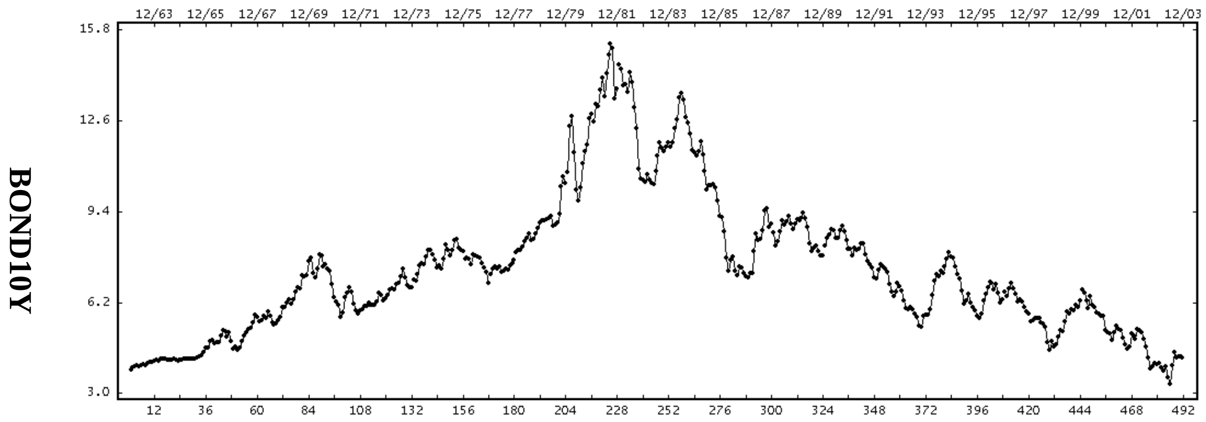
$$\begin{aligned}
\hat{C} &= 0.0053 \quad (t = 1.92) \\
\hat{\beta}_1 &= -0.3440 \quad (t = -5.20) \\
\hat{\beta}_2 &= 0.8656 \quad (t = 3.91) \\
\hat{\theta} &= -0.2480 \quad (t = -3.42) \\
\hat{\sigma} &= 0.030279
\end{aligned}$$

The above ESTIM command produces very different estimates of $\hat{\beta}_2$. By discounting the effect of the atypical data using the WESTIM command, the leading indicator has marginal influence on the S&P 500 index. In contrast, by including the atypical periods in model estimation, the leading indicator shows rather significant.

12.4 Example of Value-segmented Analysis Using the HSOLD and BON10Y Series

In this example, a transfer function model is used in conjunction with weighted estimation to study the symmetry of the elasticity estimates between interest rate increases and interest rates decreases. The two original series are presented below. The first graph is monthly sales of new homes (referred to as houses sold hereafter). The second graph is ten-year bond rates. These monthly series span from January 1963 through December 2003, a total of 512 observations for each series.





Houses sold and the 10-year bond rate exhibits a roughly inverse relationship. We are interested in studying whether an interest rate increases or decreases have similar effect (elasticity) on houses sold. The weighted estimation method can be used effectively for investigating such behavior. Since the bond rate varied greatly during the period under study, we will examine the interest rate elasticity effect across various time spans. These time spans are (1) January 1963 through December 1979 where the bond rate generally increased during the period; (2) January 1980 through December 1985 where the bond rate was unusually high; (3) January 1986 through December 2003 where the bond rate generally declined during the period; and (4) the entire span of data.

Using the LTF identification method, we obtain the following transfer function model between log transformed houses sold (HSOLD_t) and log transformed bond rate ($\mathrm{BONDRATE}_t$):

$$\nabla(\mathrm{HSOLD}_t) = \omega_1 \nabla(\mathrm{BONDRATE}_{t-1}) + (1-\theta_1 B)a_t$$

Note that the bond rate leads houses sold by one month as shown in the above model. For comparison purposes, the above model is specified using the TSMODEL command and estimated using the standard ESTIM command with all data used.

```
➔ TSMODEL HSMDL.  MODEL IS HSOLD(1)=(1)BOND10Y(1)+(1)NOISE.
➔ ESTIM HSMDL.  METHOD EXACT.
```

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  HSMDL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1
 HSOLD     RANDOM     ORIGINAL     (1-B  )
                                        1
BOND10Y    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-      VALUE      STD      T
   LABEL       NAME    DENOM.                 TRAINT                ERROR  VALUE
   1         BOND10Y   NUM.      1      1      NONE     -.5775     .0731  -7.90
   2          HSOLD     MA       1      1      NONE      .2705     .0432   6.26

EFFECTIVE NUMBER OF OBSERVATIONS . .           490
R-SQUARE . . . . . . . . . . . . . .         0.925
RESIDUAL STANDARD ERROR. . . . . . .  0.680262E-01
```

In this illustrative example, we now consider two different types of weight variables to explore the elasticity of houses sold. For the first weight variable, the weight is set to 1 if the bond rate in the previous month went down and 0 otherwise (labeled as “Rate-down”). For the second weight variable, the weight is set to 1 if the bond rate in the previous month went up and 0 otherwise (labeled as “Rate-up”). Thus by using the decrease or increase of bond rates, two value-segmented models are formed. Such models are also referred to as *threshold models* in the literature.

To accomplish the analysis, the DIFFERENCE command below is used to obtain the first-order difference of the BOND10Y series (DFBOND). The REGIME command is then employed on the DFBOND series to segment the DFBOND series based on its previous month values that are less than 0 (RATEDOWN) or greater than 0 (RATEUP).

```
➔ DIFFERENCE BOND10Y.  DFORDER 1.  NEW DFBOND.
➔ DFBOND(1)=0.0
➔ REGIME DFBOND.  THRESH 0.0.  THLAG 1. @
     INDICATORS RATEDOWN, RATEUP.
```

```
INDICATOR  VARIABLE NEGCHG   IS CREATED FOR REGIME 1 (  240 CASES SELECTED)
INDICATOR  VARIABLE POSCHG   IS CREATED FOR REGIME 2 (  251 CASES SELECTED)
```

The first twelve observations of the RATEDOWN and RATEUP indicator variables, along with BOND10Y and DFBOND, are printed below to illustrate what has been accomplished through the REGIME command.

```
➔ PRINT BOND10Y, DFBOND, RATEDOWN, RATEUP. SPAN 1,12.
```

```
VARIABLE   BOND10Y    DFBOND    RATEDOWN    RATEUP
     1       1.343      .000       .000       .000
     2       1.366      .023       .000      1.000
     3       1.369      .003       .000      1.000
     4       1.379      .010       .000      1.000
     5       1.369     -.010       .000      1.000
     6       1.384      .015      1.000       .000
     7       1.391      .007       .000      1.000
     8       1.386     -.005       .000      1.000
     9       1.406      .020      1.000       .000
    10       1.413      .007       .000      1.000
    11       1.416      .002       .000      1.000
    12       1.418      .002       .000      1.000
```

The TSMODEL command below is used to specify the models for RATEUP and RATEDOWN regimes. This is followed by the WESTIM commands to estimate the HSMDLUP model for the rate increase regime and the HSMDLDOWN for the rate decrease regime.

```
➔ TSMODEL HSMDLUP.  MODEL IS HSOLD(1)=(1)BOND10Y(1)+(1)NOISE.
➔ WESTIM HSMDLUP.  METHOD EXACT.  WEIGHT RATEUP.
```

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  HSMDLUP
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                        1
 HSOLD     RANDOM     ORIGINAL     (1-B  )
                                        1
BOND10Y    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------
```

```
  PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
    LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
   1          BOND10Y   NUM.      1      1     NONE     -.4709     .1122  -4.20
   2           HSOLD     MA       1      1     NONE      .1798     .0642   2.80

 TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           492
 EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           490
 RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.683972E-01
 RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.703243E-01
 NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           250
```

➔ TSMODEL HSMDLDOWN. MODEL IS HSOLD(1)=(1)BOND10Y(1)+(1)NOISE.
➔ WESTIM HSMDLDOWN. METHOD EXACT. WEIGHT RATEDOWN.

```
 SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  HSMDLDOWN
 ----------------------------------------------------------------------
 VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
           VARIABLE   OR CENTERED
                                         1
  HSOLD     RANDOM     ORIGINAL     (1-B  )
                                         1
 BOND10Y    RANDOM     ORIGINAL     (1-B  )
 ----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
    LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
   1          BOND10Y   NUM.      1      1     NONE     -.6750     .0895  -7.54
   2           HSOLD     MA       1      1     NONE      .3922     .0537   7.30

 TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           492
 EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           490
 RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.686248E-01
 RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.644704E-01
 NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           240
```

From the above estimation results, we find that the elasticity estimate for rate increases is -.4709; the elasticity estimate for rate decreases is -.6750; and the elasticity estimate using standard estimation methods is -.5775. These elasticity estimates are quite different. To study whether the elasticity estimates exhibit such behavior when interest rates are generally increasing (1/1963 – 12/1979), unusually high (1/1980 – 12/1985), or generally decreasing (1/1986 – 12/2003), the above analysis is repeated for these three time periods. The elasticity estimates are presented in the following summary table.

	Elasticity Estimates		
Period	**All Data**	**Rate-up**	**Rate-down**
Entire series (1/1963 – 12/2003)	-.5775	-.4709	-.6750
Increasing bond rate period (1/1963 – 12/1979)	-.3979	-.2597	-.6237
Unusually high bond rate period (1/1980 – 12/1985)	-1.1627	-1.0286	-1.2609
Decreasing bond rate period (1/1986 – 12/2003)	-.3951	-.3529	-.4525

In the above estimation results, we are particularly interested in the symmetry or asymmetry of the elasticity estimates. We find that the elasticity of houses sold is consistently greater when the interest rate change is negative (Rate-down) in the four time spans considered. We also find that the highest elasticity of houses sold, regardless of Rate-up or Rate-down regime, was during the high bond rate period.

In this chapter, general weighted estimation was discussed using the WESTIM command along with the WFORECAST command to blend weighted forecasts. It was shown how the REGIME and RECODE commands can be used to generate indicator variables that discern particular regimes for both time-segmented and value-segmented models. In the next chapter, value-segmented models (or threshold models) will be further discussed.

CHAPTER 13

THRESHOLD TIME SERIES MODELING AND FORECASTING

The Professional Edition (B) of the SCA Statistical System includes nonlinear time series modeling and forecasting capabilities. The commands TARTEST, TARXTEST, TARESTIM, TARFORECAST, THMTEST, and THMEXPLORE are documented in this chapter. Related commands including WESTIM and WFORECAST were discussed in Chapter 12. Additional information on nonlinear time series analysis can be found in Liu (2006).

In Chapter 12, a value-segmented approach was used to analyze the relationship between houses sold and long-term interest rate. The month-to-month changes in long-term interest rate were segmented into positive and negative cases using zero as the threshold value and one as the threshold lag. The model used was a threshold transfer function model with an MA(1) noise. The WESTIM command was used to estimate the parameters in the threshold transfer function models.

In this chapter, we first discuss a special case of threshold model for the univariate autoregressive case. This class of model is referred to as the threshold autoregressive (TAR) model. TAR models have been studied extensively by Tong (1978, 1983, 1990), Tong and Lim (1980), Maravell (1983), Tsay (1986, 1989), Chen and Tsay (1991, 1993), Tiao and Tsay (1994), and others.

In the next section, we shall cover the nonlinearity test of a time series, the identification of a TAR model for a time series, TAR model estimation, forecasting, and other extensions of such models using the SCA System.

13.1 Threshold Autoregressive Models

A k-regime TAR model for a time series Y_t with t=1, 2, …, n, can be written as

$$Y_t = C_0^{(i)} + \phi_1^{(i)} Y_{t-1} + \cdots + \phi_p^{(i)} Y_{t-p} + a_t^{(i)}, \qquad r_{i-1} \le Y_{t-d} < r_i \tag{13.1}$$

where i=1,2, …, k, and d is a positive integer often referred to as a threshold lag (or delay parameter). The thresholds (r_i) are non-overlapping intervals on the real line with $-\infty = r_0 < r_1 < \ldots < r_k = \infty$. The innovations $a_t^{(i)}$'s are identically and independently distributed random variables for each regime, and are independent across the regimes with potentially different variances. Even though Y_t follows a linear AR model in each regime, the overall process for (13.1) is nonlinear when there are at least two regimes with different linear models. This class of models was proposed by Tong (1978, 1983) and Tong and Lim (1980) as alternative models for describing periodic time series. It has certain features that allow it to model time series with limit cycles, jump phenomena, and amplitude dependant frequencies. The TAR model in (13.1) is also referred to as self-exciting threshold autoregressive

(SETAR) model because the AR model in each regime is determined by its own past value. Following the threshold principle, some other models, such as threshold MA models (Wecker 1981, Jolliffe and Kumar 1985), threshold ARMA models (Wang *et al.*, 1984), and adaptive spline threshold autoregression (ASTAR) models (Lewis and Stevens, 1991) have also been proposed.

Using the backshift operator B, the model in (13.1) can be written as

$$(1-\phi_1^{(i)}B-\cdots-\phi_p^{(i)}B^p)Y_t = C_0^{(i)} + a_t^{(i)}, \qquad i=1,2,\ ...,\ k \tag{13.2}$$

or

$$\phi^{(i)}(B)Y_t = C_0^{(i)} + a_t^{(i)},\ \text{with}\ \phi^{(i)}(B) = 1-\phi_1^{(i)}B-\ \cdots\ -\phi_p^{(i)}B^p \quad . \tag{13.3}$$

In the models (13.1), (13.2), and (13.3), the constant term $C_0^{(i)}$ in each regime does not have a straightforward interpretation. It is more appropriate to write (13.2) and (13.3) in the following form

$$Y_t = C^{(i)} + \frac{1}{1-\phi_1^{(i)}B-\cdots-\phi_p^{(i)}B^p}a_t^{(i)}, \qquad i=1,2,\ ...,\ k \tag{13.4}$$

or

$$Y_t = C^{(i)} + \frac{1}{\phi^{(i)}(B)}a_t^{(i)} \tag{13.5}$$

The constant term $C^{(i)}$ in (13.4) and (13.5) is then the mean for each regime. In some applications, the comparison of the mean values among regimes could be useful and even the focus of a study.

The models in (13.1) through (13.5) can be extended to multiplicative AR models as below:

$$\phi^{(i)}(B)\Phi^{(i)}(B^s)Y_t = C_0^{(i)} + a_t^{(i)}, \qquad i=1,2,\ ...,\ k \tag{13.6}$$

or

$$Y_t = C^{(i)} + \frac{1}{\phi^{(i)}(B)\Phi^{(i)}(B^s)}a_t^{(i)}, \qquad i=1,2,\ ...,\ k \tag{13.7}$$

where $\Phi^{(i)}(B^s)$ is a stationary seasonal AR polynomial in B^s.

More generally, the models in (13.6) and (13.7) can be extended to multiplicative seasonal ARIMA models as

$$\phi^{(i)}(B)\Phi^{(i)}(B^s)Y_t = C_0^{(i)} + \theta^{(i)}(B)\Theta^{(i)}(B^s)a_t^{(i)}, \qquad i=1,2,\ldots,k \tag{13.8}$$

or

$$Y_t = C^{(i)} + \frac{\theta^{(i)}(B)\Theta^{(i)}(B^s)}{\phi^{(i)}(B)\Phi^{(i)}(B^s)}a_t^{(i)}, \qquad i=1,2,\ldots,k \tag{13.9}$$

where $\theta^{(i)}(B)$ and $\Theta^{(i)}(B^s)$ are MA and seasonal MA polynomials in $B \text{ and } B^s$. In most of the literature, only TAR models in the forms of (13.1) or (13.2) are discussed. We shall also focus on this class of TAR models in this chapter.

There are two important issues involved in building a threshold model. The first is the determination of the threshold lag (d) and the corresponding threshold values. The second is the estimation of the model parameters in each regime. Assuming that the first issue has been resolved, all threshold models discussed can be estimated using the weighted estimation method discussed in Chapter 12.

The TARESTIM command in the SCA System is designed to accommodate the estimation of all models shown above. Unlike threshold AR models, certain assumptions may need to be imposed in order for the estimation of threshold MA or ARIMA models to be valid.

13.2 Nonlinearity Test

Before we pursue building a TAR model, it is prudent to check if the time series under study is nonlinear and therefore warrants consideration of a TAR model. The SCA System employs the TARTEST command that is based on the work of Tsay (1986, 1989, 1991) and others.

Tsay (1986) generalizes the procedure of Keenan (1985) and develops an F-statistic to test a very general class of nonlinear time series. Petruccelli and Davies (1986) use normalized predictive residuals to derive a *CUSUM portmanteau test* for TAR processes. Tsay (1989) combines the procedures of Keenan (1985), Tsay (1986), and Petruccelli and Davies (1986) and develops another F-statistic specifically to test for alternative TAR models. We shall refer to this particular F-test as *TAR-F test*. Tsay (1991) further extends the TAR-F test and develops a new F-test by including a larger class of alternative TAR models.

As pointed out in Tsay (1991), the power of a nonlinearity test inevitably will suffer when the class of models considered is too large. Since the class of TAR models is most useful in business and economic applications, we shall primarily use the TAR-F test for checking for nonlinearity of a time series. In addition to the increase in power of the test, Tsay's TAR-F test also provides useful information for the potential value of the threshold lag (d).

13.3 Arranged Autoregression

The TARTEST is used to obtain the threshold lag and threshold values based on the past values of the series itself. When the threshold variable is an external variable as opposed to past values of the series itself, the TARXTEST command can be used instead. The syntax for these two commands is provided in Appendix B.

The rationale and the procedure for the TAR-F test will be outlined in the following section. The TAR-F test and several other nonlinearity tests use a novel computational procedure called *arranged Autoregression* (AAR). We shall use a simple example to illustrate this procedure.

Assume that we have a time series Y_t with eight observations as below:

t	1	2	3	4	5	6	7	8
Y_t	1.61	0.32	-1.63	-1.70	-0.71	-2.86	-2.40	1.48

Let us also assume the above time series may follow a *linear* or *nonlinear* AR(1) model with zero mean which can be written as

$$Y_t = \phi_1 Y_{t-1} + a_t , \qquad t=1, 2, \cdots, 8 . \tag{13.10}$$

To obtain the estimate of ϕ_1 under a linear AR(1) model, traditionally we would regress Y_t on Y_{t-1} using the following data sequence.

t	Y_t	Y_{t-1}
1	1.61	- -
2	0.32	1.61
3	-1.63	0.32
4	-1.70	-1.63
5	-0.71	-1.70
6	-2.86	-0.71
7	-2.40	-2.86
8	1.48	-2.40

Using the seven complete pairs of data in the above table, the least squares estimate of ϕ_1 in (13.10) is 0.4106.

Now let us assume the above time series follow a TAR(1) model with d=1. We can arrange the data sequence in the above table in ascending order according to the threshold variable Y_{t-1}. Thus we have the following arranged autoregression data sequence:

t	Y_t	Y_{t-1}	$\hat{\phi}_1$
7	-2.400	-2.860	- -
8	1.480	-2.400	0.2376
5	-.710	-1.700	0.2685
4	-1.700	-1.630	0.3741
6	-2.860	-.710	0.4663
3	-1.630	0.320	0.4379
2	0.320	1.610	0.4106

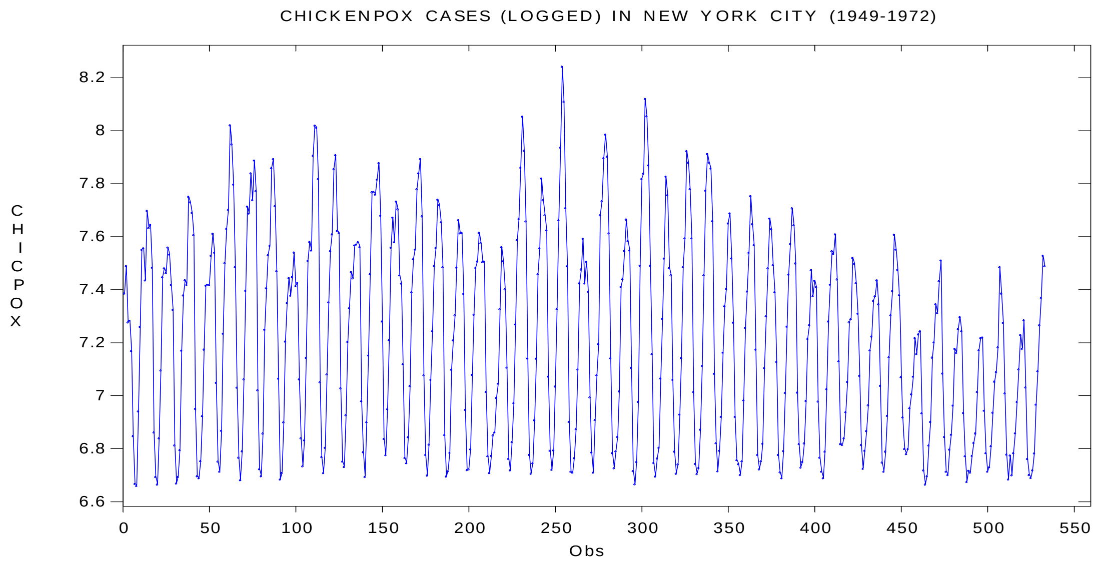
The above data sequence contains the same information as in the previous table except that the data are ordered by Y_{t-1} rather than by t. Using the arranged autoregression data, we can compute sequential estimates of ϕ_1 according to the order of Y_{t-1}. These sequential estimates are listed in the last column of the above table. It is important to note that the last sequential estimate of ϕ_1 (0.4106) is the same as the least squares estimate for the linear AR(1) model using the entire data set.

The sequential estimates of ϕ_1 (based on the ordered threshold variable) and their related statistics are very useful. They can be used to develop nonlinearity tests as well as to identify threshold values. To improve computational efficiency, recursive estimation algorithms are typically used for the computation of autoregressive coefficients. The results under recursive estimation are the same as the straightforward least squares estimates using the same data. In arranged autoregression, the data sequence can be arranged either in ascending order of the threshold variable or in descending order of the threshold variable. We refer to the latter as *reverse arranged autoregression* (RAAR). This method also provides useful information in identification of threshold values.

13.4 Example of TAR Modeling Using Chickenpox Cases in New York City

To illustrate TAR modeling capabilities in the SCA System, we analyze the number of Chickenpox cases that occurred monthly in New York City from 1949-1972 (Sugilara and May 1990). The natural log transformation is applied to the time series to stabilize the variability. This time series was analyzed using TAR models by Chen and Tsay (1993). The log transformed time series is first brought into the SCA System under the variable named CHICPOX. A time series plot of the log transformed monthly Chickenpox series is shown below. There are a total of 533 observations.

Time Series Plot of the Log Transformed Chickenpox Cases in New York City



The TARTEST command is used to examine the nonlinearity of this series using a linear AR(12) as the null hypothesis model.

➔ TARTEST CHICPOX. ARLAGS 1 TO 12. THLAGS 1 TO 12.

```
    Lag     F-statistic  Degrees of freedom
 --------   -----------  ------------------
     1        3.73837       13      443
     2        3.92756       13      443
     3        5.09289       13      443
     4        7.24443       13      443
     5        6.36462       13      443
     6        6.20812       13      443
     7        3.56343       13      443
     8        2.80965       13      443
     9        6.72298       13      443
    10       13.54124       13      443
    11       12.06614       13      443
    12        6.82576       13      443
```

From the above table, we see that the CHICPOX series shows strong nonlinearity because the F-statistics for all threshold lags are significant at both 5% and 1% levels. The F-statistics for the threshold lag at lags 10, 11, and 12 are particularly large. Because this series consists of monthly data, Chen and Tsay (1993) choose the threshold lag as 12 in their analysis.

Identification of threshold values

After the threshold lag (d) is determined, we can proceed to determine the values of the thresholds for the regimes based on the threshold variable Y_{t-d}. Tsay (1989) employs arranged autoregession and recursive estimation to obtain the sequential estimates of autoregressive parameters, their t-ratios, and corresponding estimated variances. These statistics can then be plotted against the threshold variable

Y_{t-d}. When a major change is spotted in these scatter plots, it suggests that a regime partition is needed at that threshold value. For convenience, we shall refer to this method of threshold value identification as the arranged autoregression (AAR) method even though the method incorporates ideas more than just arranged autoregression.

In preliminary analysis of the chickenpox series, the autoregressive coefficients for lags 1 and 24 are most significant. Therefore we consider the following linear autoregressive model:

$$Y_t = C_0 + \phi_1 Y_{t-1} + \phi_{24} Y_{t-24} + a_t \,. \tag{13.11}$$

To identify the threshold values for the CHICPOX series using the AAR method, we once again employ the TARTEST command. The THLAG is set to 12 and the ARLAGS are specified as 1 and 24 only. The autoregressive parameter estimates, their t-ratios, and corresponding estimated variances are saved in the SCA workspace using the HOLD subcommand.

```
➔ TARTEST CHICPOX.  THLAG 12.  ARLAGS 1, 24. @
    HOLD THVARIABLE(THVAR,THINDX),PHI(PHIMTX),TPHI(TPHIMTX), @
    SIGMAHAT(SIGMA).
```

The threshold values (in sorted order) are stored under the THVAR variable name. The PHIMTX and TPHIMTX variables store the AR estimates and their corresponding t-values in matrix form. The following SCA matrix operation separates the matrix columns into variables that can be readily used for graphing or further analysis.

```
➔ PHI1=PHIMTX(.,1)
➔ PHI24=PHIMTX(.,2)
➔ TPHI1=TPHIMTX(.,1)
➔ TPHI24=TPHIMTX(.,2)
```

Alternatively, the PICK command can be used in place of the matrix operations above to obtain columns of a matrix to create the PHI1, PHI24, TPHI1, and TPHI24 series.

Using the AAR method to obtain the estimates of ϕ_1, ϕ_{24}, and their associated statistics, scatter plots for the $\hat{\phi}_1$, $\hat{\phi}_{24}$, and their t-ratios versus the threshold variable Y_{t-12} are displayed. Note that the range of $\hat{\phi}_1$ and t-ratios of $\hat{\phi}_1$ are different from those of $\hat{\phi}_{24}$ and the t-ratios of $\hat{\phi}_{24}$. We usually prefer to use the same scale across the estimates of the model parameters (and similarly the t-ratios). In this case however, the ranges for $\hat{\phi}_1$ and $\hat{\phi}_{24}$ are very different. Therefore, different scales are used in order to display the features of the estimates for each parameter more prominently.

In the identification of threshold values, it is important to focus only on scatter plots that contain significant t-ratios. Scatter plots of insignificant AR coefficients or their t-ratios are counterproductive. Other important guidelines for the identification of threshold values will be discussed later.

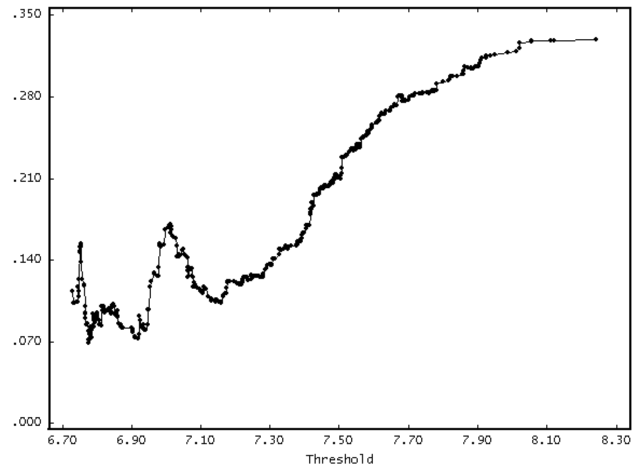
The SCA command to transfer the variables for threshold identification to the SCAGRAF applet is provided below.

➔ GRAPH THVAR, PHI1, PHI24, TPHI1, TPHI24.

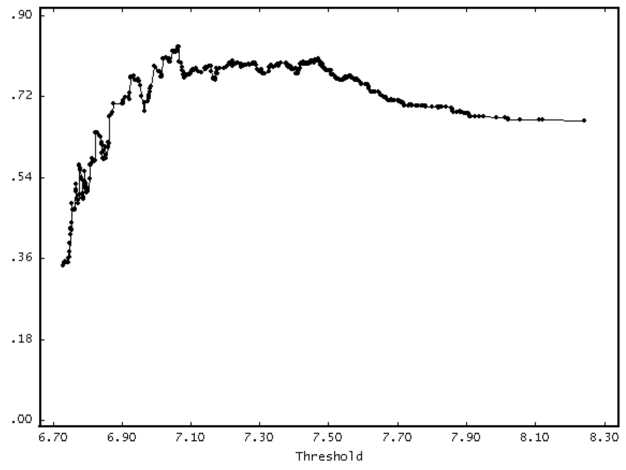
The scatter plots are then created using the menu and dialog box graphical user interface of SCAGRAF.

Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-12}
(chickenpox series with the AAR method)

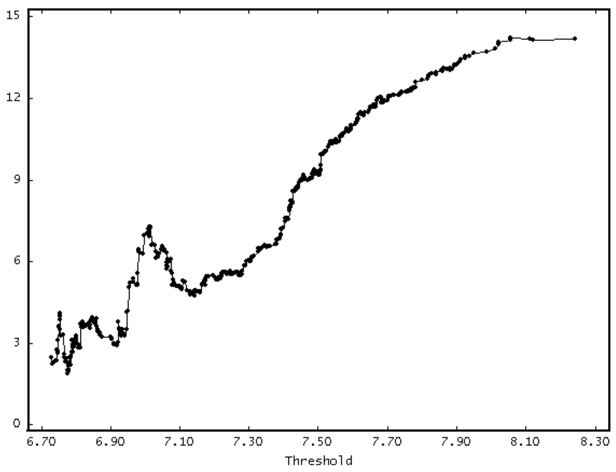
(a) Arranged autoregression estimates of ϕ_1



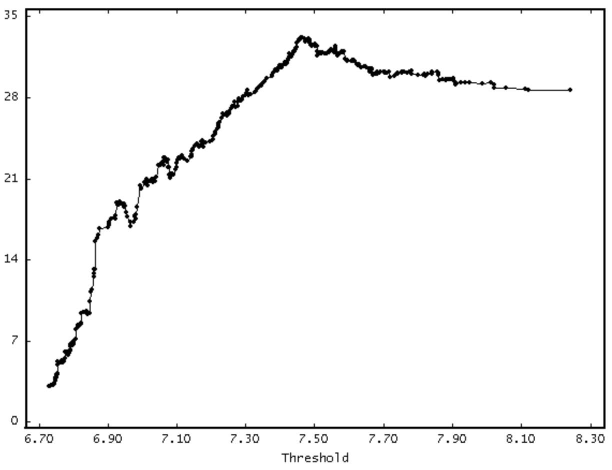
(b) Arranged autoregression estimates of ϕ_{24}



(c) t-ratios for the estimates of ϕ_1



(d) t-ratios for the estimates of ϕ_{24}



By examining the scatter plots for the ϕ_1 estimates and the corresponding t-ratios, we find potential threshold values near 6.92, 7.0, and 7.15. On the other hand, by examining the scatter plots for the ϕ_{24} estimates and the corresponding t-ratios, we find potential threshold values near 6.92 and 7.45. Because the threshold 7.0 is very close to 6.92 and because observations greater than 7.45 in the series are limited (particularly in the latter part of the time series), we may conclude the threshold values are $r_1 = 6.92$ and $r_2 = 7.15$. While these values are close, they differ from those identified by Chen and Tsay (1993). This discrepancy is due in part that the features for the thresholds in the scatter plots of the chickenpox series are not prominent. Consequently, it is somewhat difficult to exactly identify the threshold values using the AAR method in this example. We shall consider modifications to the AAR method in the next section which may facilitate easier identification of threshold values.

13.5 Modified Methods for the Identification of Threshold Values

One obvious problem in using recursive estimation of arranged autoregression (Tsay, 1989) is the recursive estimates eventually converge to those of the linear autoregressive estimates. Thus the AAR method employed in the previous section is most effective in identifying the first threshold value from the left-hand-side section (lower-end) of the threshold axis. The AAR method may start to lose effectiveness for the coefficients beyond the first threshold. This is due to the confounding (or averaging) effect on the parameter estimates as the estimation range includes more regimes. In this section, we consider two variations of the AAR method.

(A) Reverse Arranged Autoregression Method

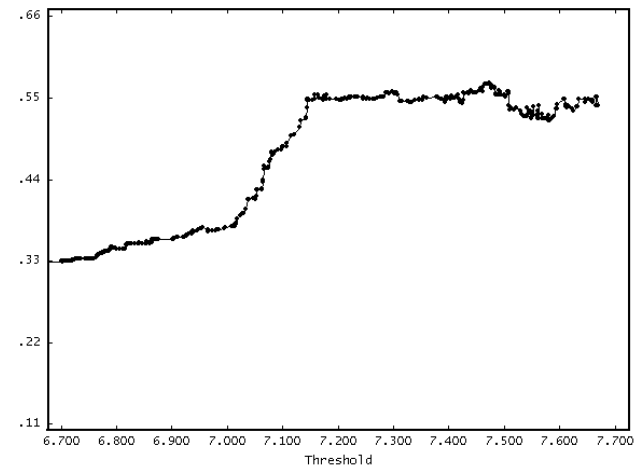
One possible way to remedy the AAR method is to consider another set of arranged autoregression estimates based on the values of the threshold variable in descending order (instead of ascending-ordered threshold variable in the original AAR method). Such an approach will allow us to more effectively identify the first threshold value from the right-hand-side section (higher-end) of the threshold axis. We shall refer to such an approach as the Reverse AAR method which is specified in the TARTEST command using the RAAR keyword in the METHOD subcommand.

```
➔ TARTEST CHICPOX.  METHOD RAAR.  THLAG 12.  ARLAGS 1, 24.                  @
     HOLD THVARIABLE(THVARR,THINDXR), PHI(PHIMTXR), TPHI(TPHIMTXR),        @
     SIGMAHAT(SIGMAR).
```

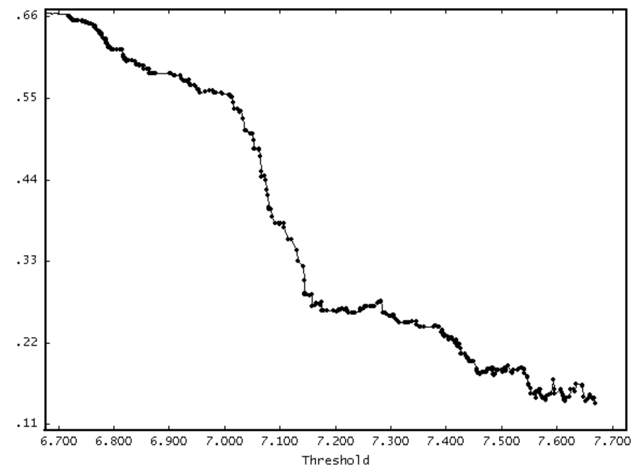
Using the Reverse AAR method, the scatter plots for $\hat{\phi}_1$, $\hat{\phi}_{24}$, and their t-ratios (versus the threshold variables) are displayed below for the chickenpox series. Based on the scatter plots of ϕ_1 estimates and their t-ratios shown in (a) and (c), we can clearly identify threshold values around 7.15 and 7.0. Similarly, when we examine the scatter plots of ϕ_{24} estimates and their t-ratios shown in (b) and (d), we can identify threshold values around 7.45, 7.15, and 7.0. The threshold 6.9 is not identified using this method since it is a threshold very close to the end of the reverse arranged autoregression sequence. In this case, the Reverse AAR method reveals more distinct features for the threshold values in comparison with the AAR method.

Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-12}
(chickenpox series with the Reverse AAR method)

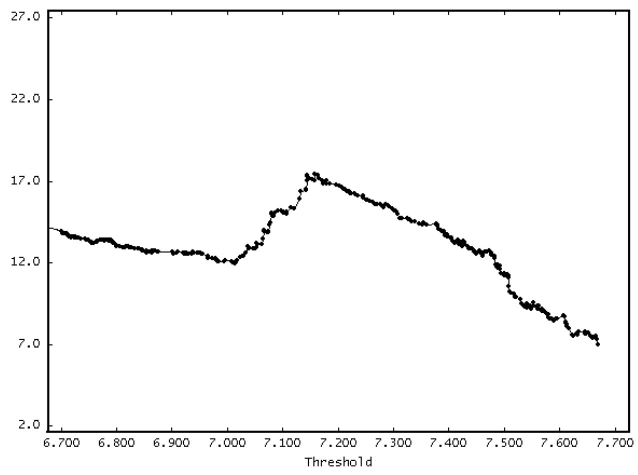
(a) Arranged autoregression estimates of ϕ_1



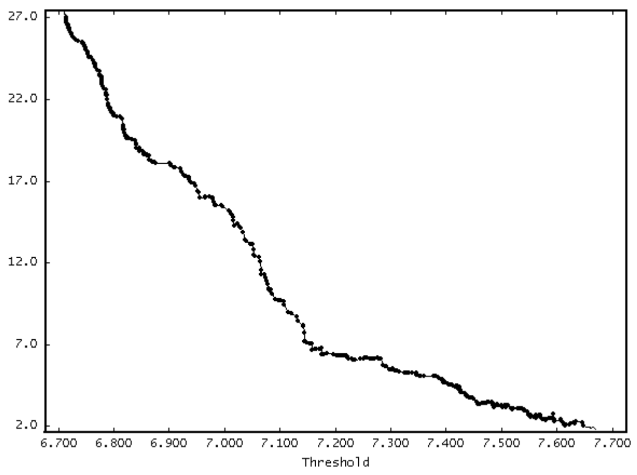
(b) Arranged autoregression estimates of ϕ_{24}



(c) t-ratios for the estimates of ϕ_1



(d) t-ratios for the estimates of ϕ_{24}



(B) Local Autoregression Method

Instead of using recursive estimation which implies all data available up to the potential threshold value are used in estimation, we may use a moving window so that only the data close to the potential threshold value are used to obtain local estimates of the model parameters and their associated statistics (e.g., t-ratios, variances, etc.). We shall refer to this method as the local autoregression (LAR) method. The LAR method is similar to the ALR method proposed in Chen and Tsay (1993). The primary difference is that the LAR method uses linear autoregression estimation for the data contained within the data window while the ALR method employs nonparametric regression estimation.

The LAR method is available through the THMEXPLORE command. The THMEXPLORE command is used to provide parameter estimates of a model according to the order of the sorted threshold variable. The threshold variable may be sorted in ascending or descending order.

A moving-window-regression estimation is used to explore the potential nonlinearity and threshold values of the model parameters. The THMEXPLORE command can be used with an autoregressive model or a transfer function model. For autoregressive models, this command provides AAR and RAAR methods in addition to the LAR method for the identification of threshold values.

In this command, the model parameters to be estimated must be specified with names (labels). For example, Y=CNST+(BETA1)X1+NOISE where CNST and BETA1 are the parameter estimate names associated with the constant and input variable X1. Furthermore, the model must be expressed in linear regression form using the TSMODEL command with white noise and without any backshift operator. All lagged variables must be created prior to using this command via the LAG command.

The parameter estimates for each data window are stored in the associated names for the parameters as vectors. The lengths of the resulting vectors for the parameter estimates are the same as the length of the output variable. The t-values for the parameter estimates are also stored for each window using the root word of the parameter label preceded by an underscore character (e.g., _CNST and _BETA1). There are estimates at the beginning of the series that are missing due to initial window size. Any parameter estimates that are unavailable (missing) are padded with the missing value code.

As a precursor to using the THMEXPLORE command, a lag 1 and lag 24 is taken on the CHICPOX series and stored as YLAG1 and YLAG24. This is performed using the LAG command.

```
➔ LAG CHICPOX.  NEW YLAG1, YLAG24.  LAGS 1, 24.
```

A model is then specified in linear regression form using the following TSMODEL command. Here, parameter labels (C, PHI1L, and PHI24L) are used in the following model specification command

```
➔ TSMODEL THMODEL.  MODEL IS @
     CHICPOX=C+(0;PHI1L)YLAG1+(0;PHI24L)YLAG24+NOISE.
```

The THMEXPLORE command is employed using the above model with a moving window size of 54 observations, and threshold lag of 12. To accommodate for missing values associated with the lagged variables, the THMEXPLORE command uses the beginning data span as the first row where all variables have non-missing values. The ending data span is similarly set to the last row where all variables have non-missing values.

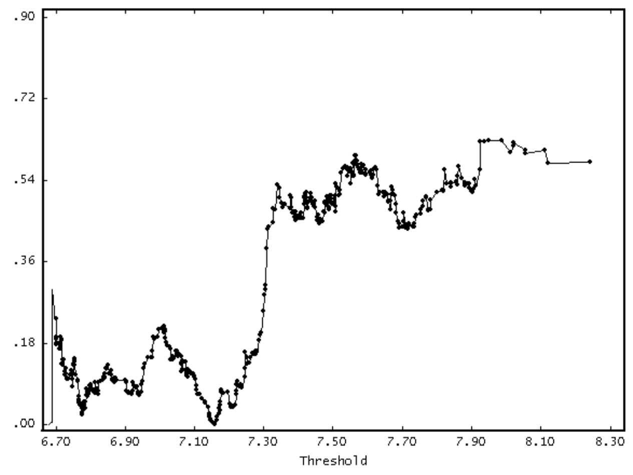
```
➔ THMEXPLORE THMODEL.  THLAG 12.  WINDOWSIZE 54. @
    HOLD THVARIABLE(THVL).
```

```
VARIABLE     THVL  STORES THE SORTED THRESHOLD VARIABLE (WITH   509 VALUES)
VARIABLE        C  STORES PARAMETER ESTIMATES (WITH   509 VALUES)
VARIABLE       _C  STORES THE t-VALUES OF THE ABOVE ESTIMATES
VARIABLE    PHI1L  STORES PARAMETER ESTIMATES (WITH   509 VALUES)
VARIABLE   _PHI1L  STORES THE t-VALUES OF THE ABOVE ESTIMATES
VARIABLE   PHI24L  STORES PARAMETER ESTIMATES (WITH   509 VALUES)
VARIABLE  _PHI24L  STORES THE t-VALUES OF THE ABOVE ESTIMATES
```

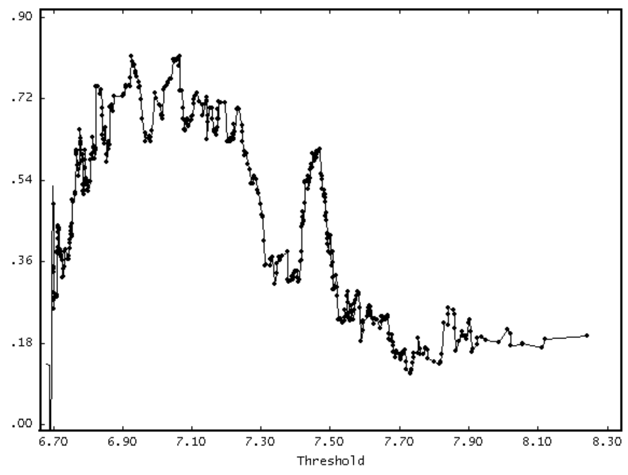
The scatter plots for the estimates of ϕ_1, ϕ_{24}, and their t-ratios versus the threshold variable Y_{t-12} are displayed next. By examining the scatter plots for the ϕ_1 estimates and their t-ratios, we find potential threshold values around 6.92, 7.0, and 7.15 as shown in (a) and (c). On the other hand, by examining the scatter plots for the ϕ_{24} estimates and their t-ratios, we find potential threshold values around 6.92 and 7.48 as shown in (d). The results under the LAR method are similar to those of the AAR and Reverse AAR methods. However, the LAR method exhibits the threshold at 6.92 more prominently than the other two methods.

Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-12} (chickenpox series with the LAR method)

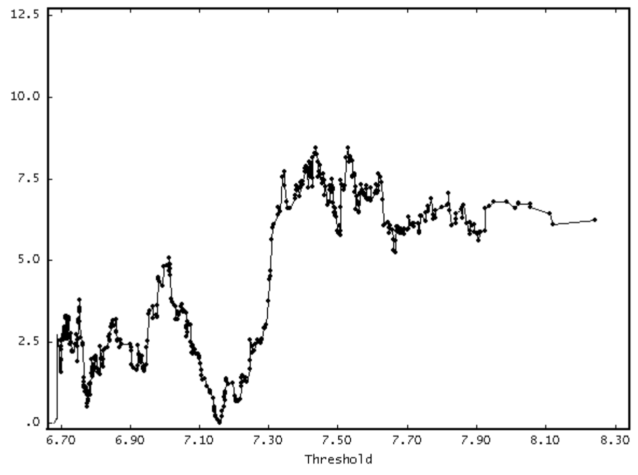
(a) Arranged autoregression estimates of ϕ_1



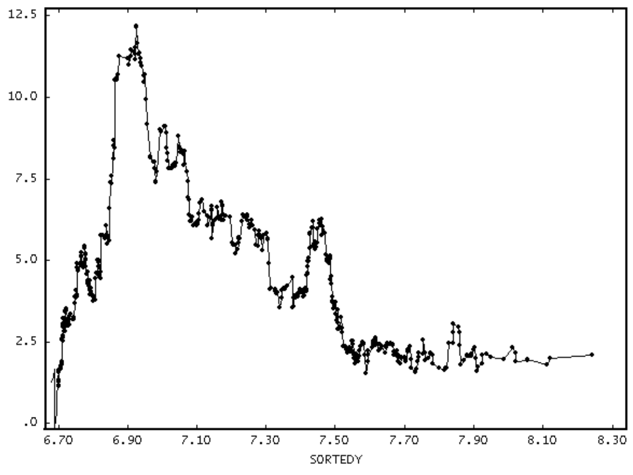
(b) Arranged autoregression estimates of ϕ_{24}



(c) t-ratios for the estimates of ϕ_1



(d) t-ratios for the estimates of ϕ_{24}



(C) Combined Use of the Methods

Following the examples discussed above, we see that all three methods for threshold value identification have their own strengths and weaknesses. The strength of the AAR and the Reverse AAR method is that both methods are effective in the identification of the first threshold value (the lower-end one for the AAR method and the higher-end one for the Reverse AAR method). In addition, we do not need to specify a data window for arranged autoregression estimation which, at times, can be complicated. When a process has only two or three regimes, the combined use of the AAR and Reverse AAR methods will fully identify the threshold values for the TAR model. In practice, a three-regime TAR model is probably more than adequate for most applications. If a process has more than three regimes, an additional application of this combined method may find two additional threshold values, resulting in a five-regime model. One weakness of these two methods is that these methods may be less effective in identifying subsequent threshold values after the first one.

The LAR method allows us to identify multiple threshold values in one pass. However, the choice of data window size may be complicated depending on the length of the series and the underlying model used for threshold value identification. In addition, the estimates of model parameters and t-ratios are more volatile than the AAR method, particularly if the window size is small. Typically, we need more data when this method is used.

In practice, it is probably prudent to use all three methods in the identification of threshold values. Ideally, we would like to see all three methods confirm one another by identifying the same or similar threshold values.

13.6 Guidelines for the Identification of Threshold Values

(1) It is advisable that extremely large outliers are pre-adjusted before embarking on threshold value identification (or nonlinearity testing). This would avoid volatile changes in the statistics to be examined. Moderate outliers need not be adjusted.

(2) When examining the scatter plots for the AAR method (where the threshold variable is sorted in ascending order), the parameter estimates at the beginning section of the scatter plot (i.e., smallest values in the threshold variable) may not be reliable due to the limited numbers of observations used in parameter estimation. This is often depicted by erratic movements in the scatter plots of parameter estimates and their t-ratios. Consequently, information shown in the beginning section of the scatter plot should be properly discounted in determining possible threshold values. A similar issue is associated with the Reverse AAR method (where the threshold variable is sorted in descending order). Here, the parameter estimates near the right-side of the scatter plot (i.e., largest values in the threshold variable) may not be reliable.

(3) The AAR scatter plots are examined for thresholds from left to right (smallest to largest value in the threshold axis). The first threshold encountered is the most reliable one. Additional thresholds detected after the initial one could still be reliable, but are often confounded by the use of the data in the earlier regime(s) in parameter estimation. Therefore, it is important to examine the Reverse AAR scatter plots for the thresholds on the higher-end of the threshold axis as well. For longer time series, the LAR method should also be considered since the presence of additional thresholds would be masked by thresholds that occur for smaller threshold values (AAR method) or larger threshold values (Reverse AAR method).

(4) Threshold candidates typically are flagged by a persistent increase (or decrease) and then leveling off in the parameter estimates or their t-ratios versus the threshold variable. The vertical range between the beginning of the threshold regime and its eventual stabilization to a new level plays a key role in determining the importance of a particular threshold candidate. It

is imperative to confirm that the parameter estimates have meaningful differences between the levels of two adjacent regimes.

(5) If there is more than one possible threshold candidate in close proximity to each other, select the threshold value that has more prominent features. It is also important to make sure that each threshold regime contains an adequate number of data points to ensure proper parameter estimation.

13.7 TAR Model Estimation

In this section, we consider three types of TAR models for the log transformed time series CHICPOX. The first model type is a conventional TAR model, replicating the work of Chen and Tsay (1993). The second model type is a multiplicative AR model, and the third model type is a "threshold ARIMA" model.

In the paper by Chen and Tsay (1993), the regimes $Y_{t-12} < 6.85$, $6.85 \le Y_{t-12} < 7.21$, and $Y_{t-12} > 7.21$ are identified with the underlying model (13.11) using the Functional-Coefficient Autoregression (FAR) approach. In the previous section, we have identified regimes (and hence threshold values) that are very close to the above values. For comparison purpose, the threshold values identified by Chen and Tsay (1993) will be used to illustrate TAR model estimation and forecasting.

<u>**Model Type 1**</u>

In Chen and Tsay (1993), the following TAR model with threshold lag (delay parameter) of 12 is considered.

$$\left(1-\phi_1^{(1)}B-\phi_3^{(1)}B^3-\phi_9^{(1)}B^9-\phi_{24}^{(1)}B^{24}\right)Y_t=\phi_0^{(1)}+a_t^{(1)} \qquad \text{if } Y_{t\text{-}12}<6.85,$$

$$\left(1-\phi_1^{(2)}B-\phi_2^{(2)}B^2-\phi_9^{(2)}B^9-\phi_{24}^{(2)}B^{24}\right)Y_t=\phi_0^{(2)}+a_t^{(2)} \qquad \text{if } 6.85 \le Y_{t\text{-}12}<7.21,$$

$$\left(1-\phi_1^{(3)}B-\phi_2^{(3)}B^2-\phi_8^{(3)}B^8-\phi_9^{(3)}B^9\right)Y_t=\phi_0^{(3)}+a_t^{(3)} \qquad \text{if } Y_{t\text{-}12}\ge 7.21,$$

The SCA System provides the REGIME command to generate the threshold indicator variables. The threshold indicator variables consist of 0's and 1's, where 1 indicates that an observation is in a specific regime and 0 otherwise. The threshold lag is specified in the REGIME command using the THLAG subcommand. In this example, the threshold lag is specified as 12. Two values, 6.85 and 7.21, are specified in the THRESHOLD subcommand, which define the three regimes. The INDICATORS subcommand is used to specify the variables that will store the regime information. Here, REGIME1 represents the regime where $Y_{t-12} < 6.85$, REGIME2 represents the regime where $6.85 \le Y_{t-12} < 7.21$, and REGIME3 represents the regime where $Y_{t-12} \ge 7.21$.

The FINDICATOR subcommand is used to hold the extended part of the threshold indicator. The FINDICATOR subcommand should be specified if we intend to perform TAR model forecasting (to be described later). The REGIME command is specified below.

```
➔ REGIME CHICPOX.  THRESHOLD 6.85, 7.21.  THLAG 12.     @
    INDICATORS REGIME1, REGIME2, REGIME3.  FINDICATOR  FI1, FI2, FI3.
```

```
INDICATOR  VARIABLE REGIME1  IS CREATED FOR REGIME  1
FINDICATOR VARIABLE FI1      IS CREATED FOR REGIME  1 FOR FORECASTING
INDICATOR  VARIABLE REGIME2  IS CREATED FOR REGIME  2
FINDICATOR VARIABLE FI2      IS CREATED FOR REGIME  2 FOR FORECASTING
INDICATOR  VARIABLE REGIME3  IS CREATED FOR REGIME  3
FINDICATOR VARIABLE FI3      IS CREATED FOR REGIME  3 FOR FORECASTING
```

Once the threshold indicators are constructed, we can now specify the TAR models for the three regimes considered in this example. The TAR models are specified using the TSMODEL command and are estimated using the TARESTIM command. An individual model is specified for each regime. The model information is stored in the SCA System as MODEL1, MODEL2 and MODEL3 (see the TSMODEL command). The SCA commands and output are presented below

Regime 1: $Y_{t-12} < 6.85$

```
➔ TSMODEL MODEL1.  MODEL IS (1,3,9,24)CHICPOX=C1+NOISE.
➔ TARESTIM MODEL1.  REGIME REGIME1.
```

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

CHICPOX    RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
  1 C1                 CNST      1      0     NONE     2.6329      .4391   6.00
  2          CHICPOX    AR       1      1     NONE      .2848      .0364   7.83
  3          CHICPOX    AR       1      3     NONE     -.0984      .0173  -5.68
  4          CHICPOX    AR       1      9     NONE      .0730      .0182   4.01
  5          CHICPOX    AR       1     24     NONE      .3517      .0630   5.58

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           509
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.250641E+00
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.426755E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           135
```

Regime 2: $6.85 \le Y_{t-12} < 7.21$

- ➔ TSMODEL MODEL2. MODEL IS (1,2,9,24)CHICPOX=C2+NOISE.
- ➔ TARESTIM MODEL2. REGIME REGIME2.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL2
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

CHICPOX    RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1 C2                 CNST      1      0     NONE      .7203      .6746   1.07
  2         CHICPOX    AR        1      1     NONE      .8700      .1221   7.12
  3         CHICPOX    AR        1      2     NONE     -.5126      .0989  -5.18
  4         CHICPOX    AR        1      9     NONE      .1175      .0438   2.68
  5         CHICPOX    AR        1     24     NONE      .4157      .0708   5.87

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           509
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.141528E+00
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.981392E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           117
```

Regime 3: $Y_{t-12} \ge 7.21$

- ➔ TSMODEL MODEL3. MODEL IS (1,2,8,9,24)CHICPOX=C3+NOISE.
- ➔ TARESTIM MODEL3. REGIME REGIME3.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL3
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

CHICPOX    RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE
  1 C3                 CNST      1      0     NONE      .2300      .3551    .65
  2         CHICPOX    AR        1      1     NONE      .8511      .0574  14.83
  3         CHICPOX    AR        1      2     NONE     -.1454      .0519  -2.80
  4         CHICPOX    AR        1      8     NONE     -.1309      .0377  -3.48
  5         CHICPOX    AR        1      9     NONE      .2559      .0350   7.32
  6         CHICPOX    AR        1     24     NONE      .1444      .0362   3.99

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           509
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.137159E+00
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.994496E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           25
```

Conditional least squares estimation is used to estimate the TAR models in the SCA System. The results produced by the SCA System are the same as those presented in Chen and Tsay (1993). SCA estimation results shown after each TARESTIM command are summarized in the table below.

$\hat{\phi}_0^{(1)} = 2.63$	$\hat{\phi}_1^{(1)} = 0.28$	$\hat{\phi}_3^{(1)} = -0.10$	$\hat{\phi}_9^{(1)} = 0.07$	$\hat{\phi}_{24}^{(1)} = 0.35$		$\hat{\sigma}_1^{(1)} = 0.04268$
$\hat{\phi}_0^{(2)} = 0.72$	$\hat{\phi}_1^{(2)} = 0.87$	$\hat{\phi}_2^{(2)} = -0.51$	$\hat{\phi}_9^{(2)} = 0.12$	$\hat{\phi}_{24}^{(2)} = 0.42$		$\hat{\sigma}_1^{(2)} = 0.09814$
$\hat{\phi}_0^{(3)} = 0.23$	$\hat{\phi}_1^{(3)} = 0.85$	$\hat{\phi}_2^{(3)} = -0.15$	$\hat{\phi}_8^{(3)} = -.13$	$\hat{\phi}_9^{(3)} = 0.26$	$\hat{\phi}_{24}^{(3)} = 0.14$	$\hat{\sigma}_1^{(3)} = 0.09945$

13.8 Forecasting

After the regime models (MODEL1 - MODEL3) are estimated, forecasts can be generated for each model up to the threshold lag (d) using the TARFORECAST command. The threshold lag (delay parameter) is 12 in this example and we will use the thresholds (6.85 and 7.21) reported by Chen and Tsay (1993) in this illustration.

We first input the threshold information into a variable THRSHINFO. The first number is the threshold lag followed by the threshold values. The threshold information is then provided to the TARFORECAST command.

```
➔ INPUT THRSHINFO.
➔   12   6.85   7.21
➔ END OF DATA
```

Below, the TARFORECAST command is specified. The NOFS subcommand is used to control the number of forecasts to generate. It is important to note that the number of forecasts cannot exceed the threshold lag value; in this case 12.

```
➔ TARFORECAST MODEL1,MODEL2,MODEL3.  THRESHOLD THRSHINFO.  NOFS 12.
```

```
---------------------------------------------------
MODEL MODEL1  :    12 FORECASTS, BEGINNING AT  533
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
  534      6.9954      0.0427
  535      6.7595      0.0444
  536      6.6678      0.0445
  537      6.7357      0.0446
  538      6.7613      0.0447
  539      6.8195      0.0447
  540      6.8635      0.0447
  541      6.9267      0.0447
  542      6.9792      0.0447
  543      6.9993      0.0448
  544      6.9633      0.0448
  545      6.9790      0.0448
---------------------------------------------------
 MODEL MODEL2  :    12 FORECASTS, BEGINNING AT  533
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
  534      7.0784      0.0981
  535      6.6475      0.1301
  536      6.4522      0.1323
  537      6.5626      0.1342
  538      6.7425      0.1381
  539      6.8976      0.1390
  540      6.9836      0.1390
  541      7.0470      0.1394
  542      7.1042      0.1396
  543      7.1270      0.1402
  544      7.0452      0.1413
  545      6.9842      0.1418

---------------------------------------------------
MODEL MODEL3  :    12 FORECASTS, BEGINNING AT  533
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
  534      7.3549      0.0994
  535      7.2106      0.1306
  536      7.0863      0.1427
  537      7.0452      0.1474
  538      7.0272      0.1491
  539      7.0605      0.1498
  540      7.1081      0.1500
  541      7.2067      0.1501
  542      7.3090      0.1505
  543      7.3846      0.1505
  544      7.4059      0.1519
  545      7.4022      0.1541

---------------------------------------------------
COMBINED FORECASTS:   12 FORECASTS, BEGINNING AT  533
---------------------------------------------------
 TIME    FORECAST   STD. ERROR   ACTUAL IF KNOWN
  534      7.0784      0.0981
  535      6.7595      0.0444
  536      6.6678      0.0445
  537      6.7357      0.0446
  538      6.7613      0.0447
  539      6.8195      0.0447
  540      6.9836      0.1390
  541      7.0470      0.1394
  542      7.3090      0.1505
  543      7.3846      0.1505
  544      7.4059      0.1519
  545      7.4022      0.1541
```

In the above TARFORECAST specification, the regime indicator variables for the forecasts are constructed internally based on the threshold information provided. In some situations, it may be desirable to provide the regime indicator variables for the forecasts directly. The TARFORECAST command allows an alternative specification using the WEIGHT subcommand. When the WEIGHT subcommand is used, more flexibility is possible (e.g., smoothing forecasts for regime transition periods). The FI1, FI2, and FI3 variables that were generated using the earlier REGIME command can now be used.

➔ TARFORECAST MODEL1,MODEL2,MODEL3. WEIGHT FI1,FI2,FI3. NOFS 12.

The output from this command is not shown since it is the same as the as the TARFORECAST command using the THRESHOLD subcommand above.

Model Type 2

Having replicated the TAR estimation results of Chen and Tsay (1993), we now deviate from the conventional TAR modeling approach and consider a multiplicative autoregressive model. The same threshold lag and regime specifications from the first approach are continued to be used here.

The IARIMA command which provides automatic ARIMA model identification is used to examine the overall behavior of the CHICPOX series without consideration of different regimes. The command and output are shown below.

➔ IARIMA CHICPOX. SEASON 12.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL -- UTSMODEL
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                       12
CHICPOX    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
   1          CHICPOX    MA       1     12     NONE      .7300     .0306  23.83
   2          CHICPOX   D-AR      1      1     NONE      .7902     .0272  29.03

TOTAL NUMBER OF OBSERVATIONS . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS . .           520
RESIDUAL STANDARD ERROR. . . . . . .  0.904489E-01
```

Upon reviewing the model identified by the IARIMA command, it shows that the seasonal behavior of the time series is represented by the model

$$\left(1-B^{12}\right)Y_t = \left(1-\Theta_1 B^{12}\right)a_t \tag{13.12}$$

which can be approximated by a seasonal AR(2) model. Therefore, the ARIMA model identified by the IARIMA command can be written as

$$Y_t = C + \frac{1}{(1-\phi_1 B)\left(1-\phi_{12}B^{12}-\phi_{24}B^{24}\right)} a_t \tag{13.13}$$

We shall use the model in (13.13) for the three regimes considered in this example. The multiplicative TAR model and regimes are

$$\begin{aligned}
Y_t &= C^{(1)} + \frac{1}{\left(1-\phi_1^{(1)}B\right)\left(1-\phi_{12}^{(1)}B^{12}-\phi_{24}^{(1)}B^{24}\right)} a_t^{(1)} && \text{if } Y_{t-12} < 6.85,\\
Y_t &= C^{(2)} + \frac{1}{\left(1-\phi_1^{(2)}B\right)\left(1-\phi_{12}^{(2)}B^{12}-\phi_{24}^{(2)}B^{24}\right)} a_t^{(2)} && \text{if } 6.85 \le Y_{t-12} < 7.21,\\
Y_t &= C^{(3)} + \frac{1}{\left(1-\phi_1^{(3)}B\right)\left(1-\phi_{12}^{(3)}B^{12}-\phi_{24}^{(3)}B^{24}\right)} a_t^{(3)} && \text{if } Y_{t-12} \ge 7.21,
\end{aligned} \tag{13.14}$$

In the above multiplicative TAR model, all autoregressive polynomials are specified on the right-hand side of the model. By doing so, the constant terms in the model $C^{(i)}$ is the mean of the associated regime, thus bringing more intuitive meaning to the parameter estimates. This is different from the conventional TAR models, where the constant term $\phi_0^{(i)}$ in each regime has no particular meaning.

The models are specified and estimated in the SCA System using the TSMODEL and TARESTIM commands. Following the same steps taken in Approach 1, the SCA commands and output are presented below.

Regime 1: $Y_{t-12} < 6.85$

```
➔ TSMODEL MODEL1.   MODEL IS  CHICPOX=C1+1/(1)(12,24)NOISE.
➔ TARESTIM  MODEL1.  REGIME REGIME1.
```

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
--------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

CHICPOX    RANDOM     ORIGINAL     NONE
--------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-      VALUE      STD      T
   LABEL       NAME    DENOM.                 TRAINT                ERROR  VALUE
  1 C1                 CNST      1      0     NONE     6.7285      .0329 204.67
  2          CHICPOX   D-AR      1      1     NONE      .2932      .0528   5.55
  3          CHICPOX   D-AR      2     12     NONE      .2712      .1003   2.71
  4          CHICPOX   D-AR      2     24     NONE      .5492      .0698   7.87

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .            533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .            508
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.132016E+00
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.454842E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .            135
```

Regime 2: $6.85 \le Y_{t-12} < 7.21$

➔ TSMODEL MODEL2. MODEL IS CHICPOX=C2+1/(1)(12,24)NOISE.
➔ TARESTIM MODEL2. REGIME REGIME2.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL2
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

CHICPOX    RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT              ERROR  VALUE
  1 C2                 CNST      1      0     NONE     7.0727      .3415  20.71
  2         CHICPOX   D-AR      1      1     NONE      .5792      .0874   6.63
  3         CHICPOX   D-AR      2     12     NONE      .3794      .0995   3.81
  4         CHICPOX   D-AR      2     24     NONE      .5536      .0881   6.29

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           508
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.972547E-01
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.104459E+00
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           117
```

Regime 3: $Y_{t-12} \ge 7.21$

➔ TSMODEL MODEL3. MODEL IS CHICPOX=C3+1/(1)(12,24)NOISE.
➔ TARESTIM MODEL3. REGIME REGIME3.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL3
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

CHICPOX    RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT              ERROR  VALUE
  1 C3                 CNST      1      0     NONE     7.7169      .1232  62.64
  2         CHICPOX   D-AR      1      1     NONE      .7926      .0355  22.34
  3         CHICPOX   D-AR      2     12     NONE      .2802      .0530   5.28
  4         CHICPOX   D-AR      2     24     NONE      .4525      .0521   8.69

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           508
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.105704E+00
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.100817E+00
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           256
```

The SCA estimation results shown after each TARESTIM command are summarized in the table below for convenience.

$\hat{C}^{(1)} = 6.73$ $\hat{\phi}_1^{(1)} = .29$ $\hat{\phi}_{12}^{(1)} = .27$ $\hat{\phi}_{24}^{(1)} = .55$ $\hat{\sigma}_2^{(1)} = .04548$

$\hat{C}^{(2)} = 7.07$ $\hat{\phi}_1^{(2)} = .58$ $\hat{\phi}_{12}^{(2)} = .38$ $\hat{\phi}_{24}^{(2)} = .55$ $\hat{\sigma}_2^{(2)} = .10446$

$\hat{C}^{(3)} = 7.72$ $\hat{\phi}_1^{(3)} = .79$ $\hat{\phi}_{12}^{(3)} = .28$ $\hat{\phi}_{24}^{(3)} = .45$ $\hat{\sigma}_2^{(3)} = .10082$

The $\hat{C}^{(i)}$ value represents the mean of the i-th regime. Comparing the results between Approach 1 and 2, we find that the residual standard errors $\hat{\sigma}^{(i)}$'s associated with Approach 2 are somewhat larger than the conventional TAR models in Approach 1. However, the results are still quite comparable. This is true especially when considering that Approach 2 uses fewer parameters, resulting in a more parsimonious model. Also in Approach 1, several AR parameter estimates are quite small even though they are statistically significant.

Model Type 3

Finally, we consider a model that deviates even further from the conventional TAR modeling approach. Here we shall use the model which was identified with the IARIMA command. This model includes a seasonal differencing operator as well as a seasonal MA parameter and a non-seasonal AR parameter.

The same model specification and estimation steps that were conducted for the two prior approaches are carried out below. Afterward, we examine the residual standard errors of all three approaches presented in this section.

Regime 1: $Y_{t-12} < 6.85$

➔ TSMODEL MODEL1. MODEL IS CHICPOX(12)=(12)/(1)NOISE.
➔ TARESTIM MODEL1. REGIME REGIME1.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                       12
CHICPOX    RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD      T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
  1          CHICPOX    MA       1     12      NONE      .6666     .0472  14.11
  2          CHICPOX   D-AR      1      1      NONE      .3296     .0533   6.19

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           520
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.112355E+00
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.436301E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           137
```

Regime 2: $6.85 \le Y_{t-12} < 7.21$

➔ TSMODEL MODEL2. MODEL IS CHICPOX(12)=(12)/(1)NOISE.
➔ TARESTIM MODEL2. REGIME REGIME2.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL2
------------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                       12
CHICPOX    RANDOM     ORIGINAL     (1-B  )
------------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
  1          CHICPOX    MA       1     12     NONE      .7477      .0583  12.83
  2          CHICPOX   D-AR      1      1     NONE      .7081      .0749   9.45

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           520
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.913542E-01
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.953122E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           120
```

Regime 3: $Y_{t-12} \ge 7.21$

➔ TSMODEL MODEL3. MODEL IS CHICPOX(12)=(12)/(1)NOISE.
➔ TARESTIM MODEL3. REGIME REGIME3.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL3
------------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED
                                       12
CHICPOX    RANDOM     ORIGINAL     (1-B  )
------------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT               ERROR  VALUE
  1          CHICPOX    MA       1     12     NONE      .7410      .0434  17.06
  2          CHICPOX   D-AR      1      1     NONE      .8443      .0343  24.63

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           533
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           520
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.907950E-01
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.101654E+00
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .           263
```

The SCA estimation results shown after each TARESTIM command are summarized in the table below for convenience.

$\hat{\phi}_1^{(1)} = .33 \qquad \hat{\Theta}_{12}^{(1)} = .67 \qquad \hat{\sigma}_3^{(1)} = .04363$

$\hat{\phi}_1^{(2)} = .71 \qquad \hat{\Theta}_{12}^{(2)} = .75 \qquad \hat{\sigma}_3^{(2)} = .09531$

$\hat{\phi}_1^{(3)} = .84 \qquad \hat{\Theta}_{12}^{(3)} = .74 \qquad \hat{\sigma}_3^{(3)} = .10165$

Upon examining the residual standard errors of the three regimes, we find that the residual standard errors under Model Type 3 are smaller or comparable to the TAR model used by Chen and Tsay (1993) while the number of parameters was reduced by more than half.

In summary, three types of threshold models are presented in this section to address nonlinearity in a time series caused by the presence of multiple regimes in the time series data. The first model type considered was a conventional TAR model that was presented by Chen and Tsay (1993) in their analysis of logged monthly Chickenpox cases in New York City. Their estimation results were replicated using the SCA System in Model Type 1. A second model type was then considered using a multiplicative AR model. In Model Type 2, we specified the autoregressive operators on the right-hand side of the model, giving more meaning to the constant term in the models. The final model type considered was a general threshold ARIMA model. The residual standard errors and the number of parameters in each model (p) for the three threshold model types are presented in the table below.

Model Type	**Regime 1**		**Regime 2**		**Regime 3**	
	p	RSE	p	RSE	p	RSE
#1	5	0.04268	5	0.09814	6	0.09945
#2	4	0.04548	4	0.10446	4	0.10082
#3	2	0.04363	2	0.09531	2	0.10165

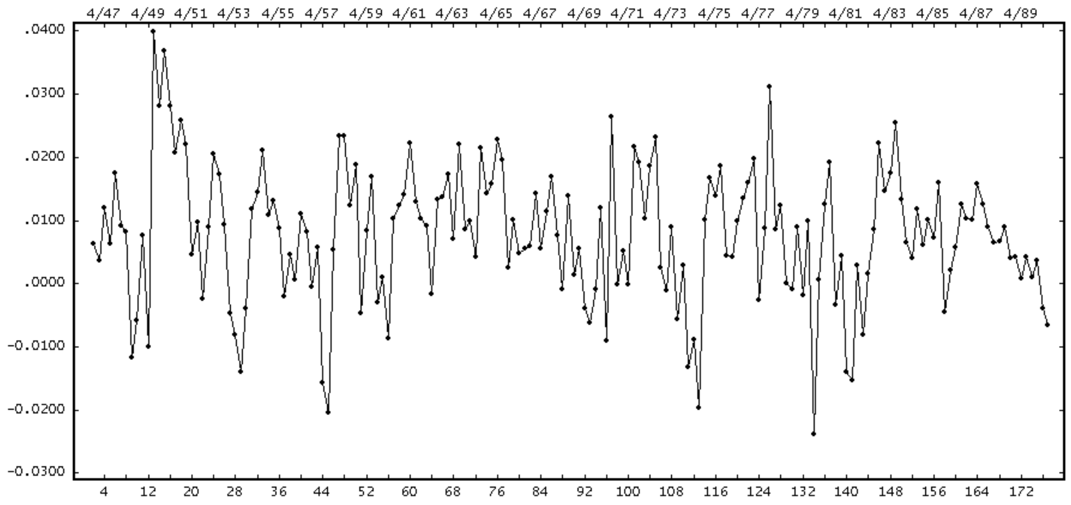
13.9 Example of TAR Modeling Using the U.S. Real GNP

In this section, a TAR model is used to analyze the quarterly U.S. real GNP series (in 1982 dollars) between the first quarter of 1947 and the first quarter of 1991. Tiao and Tsay (1994) study the growth rate of the U.S. real GNP series (denoted as DGNP_t below) with

$$\text{DGNP}_t = \ell n(\text{GNP}_t) - \ell n(\text{GNP}_{t-1}) = (1-B)\ell n(\text{GNP}_t) \tag{13.15}$$

where GNP_t is the quarterly real GNP in the t-th quarter and DGNP_t is the growth rate between the successive quarters. Due to differencing, the series DGNP_t has 176 observations and is displayed below.

Growth rate of U.S. quarterly real GNP (2/1947 – 1/1991)



Using the entire time series, we find that the GNP growth rate (DGNP_t) can be represented by a MA(2) or an AR(1) model. For purpose of TAR modeling, we consider the following slightly over-parameterized AR(2) model:

$$\mathrm{DGNP}_t = C_0 + \phi_1 \mathrm{DGNP}_{t-1} + \phi_2 \mathrm{DGNP}_{t-2} + a_t . \tag{13.16}$$

The estimates for the parameters of the above model are

$$\hat{C}_0 = 0.0041 \text{ (t=4.13)}, \quad \hat{\phi}_1 = 0.3304 \text{ (t=4.39)}, \quad \hat{\phi}_2 = 0.1344 \text{ (t=1.78)}, \quad \text{and } \hat{\sigma}_a = 0.00986 \tag{13.17}$$

Using this linear AR(2) as the null hypothesis model, the statistics for the TAR-F test for possible nonlinear threshold lags (d) are computed using the TARTEST command. Here, we consider threshold lags from 1 to 6.

➔ TARTEST DGNP. ARLAGS 1, 2. THLAGS 1 TO 6.

```
    Lag     F-statistic  Degrees of freedom
  --------   -----------  ------------------
     1       0.36586        3     152
     2       3.16414        3     152
     3       2.55177        3     151
     4       2.65051        3     150
     5       1.70234        3     149
     6       1.79730        3     148
```

The above table shows that the linear AR(2) model should be rejected because the F-statistic at d=2 (the largest) is significant at 5% level. Thus a TAR model with d=2 should be considered. If multiple d values are significant, we usually choose the d that has the largest corresponding F-value. In some applications we may select a threshold lag based on additional rationale. For example, if a few significant F statistics are clustered around a seasonal lag, the seasonal lag may be selected instead of a more significant threshold lag due to the seasonality of the particular series.

To identify the threshold values for the DGNP series using the AAR method, the TARTEST command is employed. The THLAG is set to 2 and the ARLAGS are specified as 1 and 2. The autoregressive parameter estimates, their t-ratios, and corresponding estimated variances are saved in the SCA workspace using the HOLD subcommand.

```
➔  TARTEST DGNP.  THLAG 2.  ARLAGS 1 TO 2.                              @
     HOLD THVARIABLE(THV,THINDX), PHI(PHIMTX), TPHI(TPHIMTX),           @
     SIGMAHAT(SIGMA).
```

The threshold values (in sorted order) are stored under the THV variable name. The PHIMTX and TPHIMTX variables store the AR estimates and their corresponding t-values in matrix form. The following SCA matrix operation separates the matrix columns into variables that can be readily used for graphing or further analysis.

```
➔  PHI1=PHIMTX(.,1)
➔  PHI2=PHIMTX(.,2)
➔  TPHI1=TPHIMTX(.,1)
➔  TPHI2=TPHIMTX(.,2)
```

Alternatively, the PICK command can be used in place of the matrix operations above to copy columns of a matrix to create the PHI1, PHI2, TPHI1, and TPHI2 series.

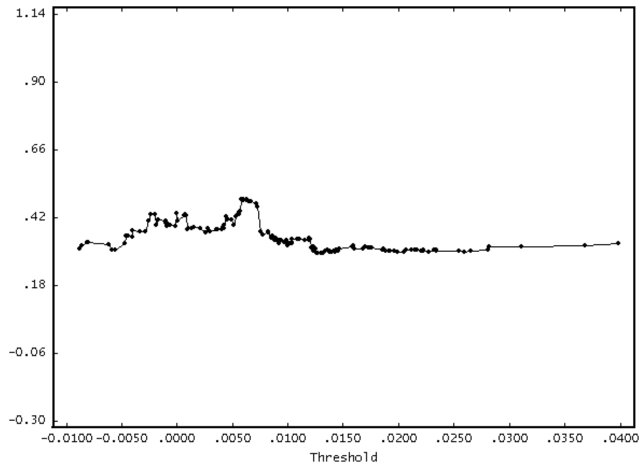
The scatter plots for the estimates (PHI1 and PHI2), and their t-ratios (TPHI1 and TPHI2) versus the threshold variable THV (i.e., DGNP_{t-2}) are created using the GRAPH command. The data is transferred to SCAGRAF using the command

```
➔  GRAPH THV, PHI1, PHI2, TPHI1, TPHI2.
```

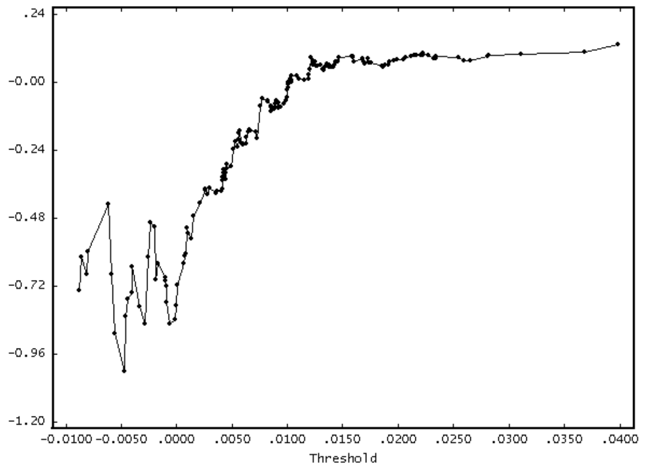
By examining the scatter plots we find that the estimates of ϕ_1 have little variation. On the contrary, the estimates of ϕ_2 vary greatly (between -1.019 and 0.134). Therefore, we shall use the estimates of ϕ_2 and the corresponding t-ratios for identification of threshold values. From (b) and (d), we find the ϕ_2 estimates and their t-ratios change their direction from descending to ascending around $Y_{t-2} = 0$. This suggests that data can be partitioned into two regimes with a threshold at $Y_{t-2} = 0.0$. However, if we also use ϕ_1 estimates and their t-ratios to identify threshold values, we find an additional threshold around $Y_{t-2} = 0.007$, as shown in (a) and (b). We should keep in mind that this threshold may not be meaningful because the ϕ_1 estimates have little variation between regimes separated by this threshold, as shown in (a).

Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-2} (GNP series with the AAR method)

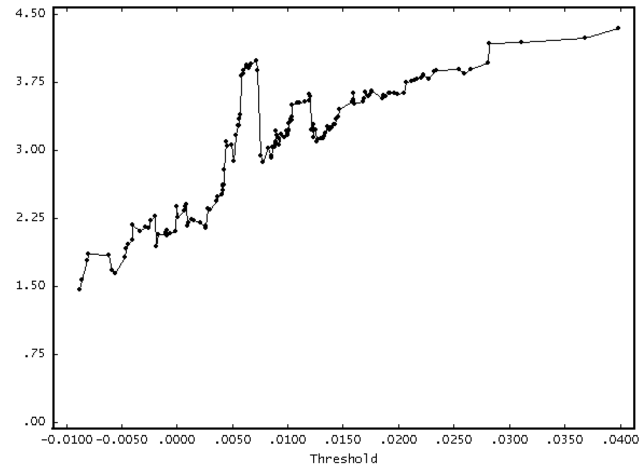
(a) Arranged autoregression estimates of ϕ_1



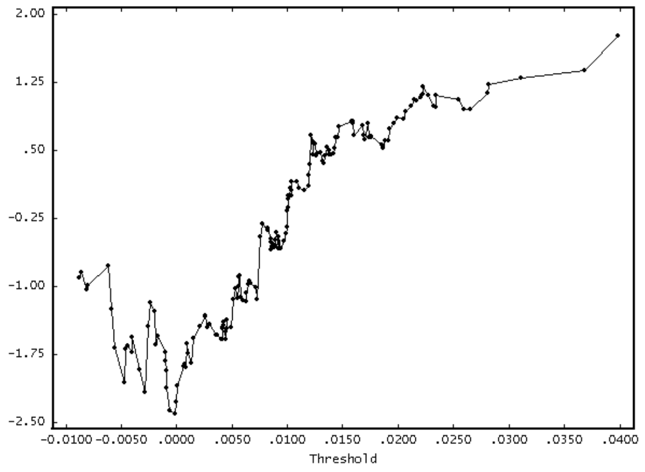
(b) Arranged autoregression estimates of ϕ_2



(c) t-ratios for the estimates of ϕ_1



(d) t-ratios for the estimates of ϕ_2



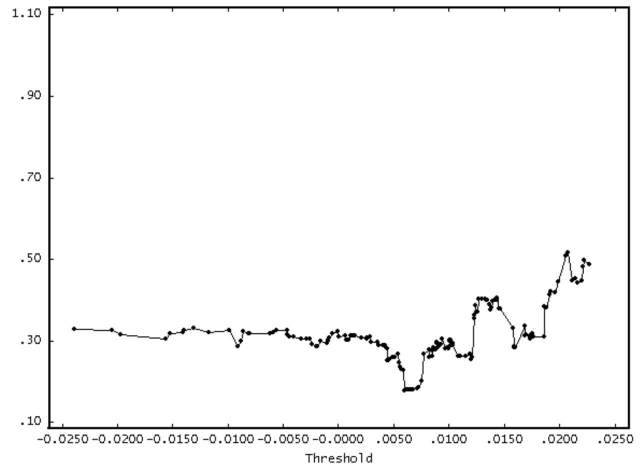
The Reverse AAR method is also applied on the GNP series. Instead of searching for threshold values from left-to-right on the threshold axis, we search for threshold values from right-to-left on the threshold axis when the Reverse AAR method is used. The following TARTEST command is specified using the RAAR method.

```
➔ TARTEST DGNP.  THLAGS 2.   ARLAGS 1, 2.  HOLD                         @
    THVARIABLE(THVR,THINDXR), PHI(PHIMTXR),TPHI(TPHIMTXR), @
    SIGMAHAT(SIGMAR).
```

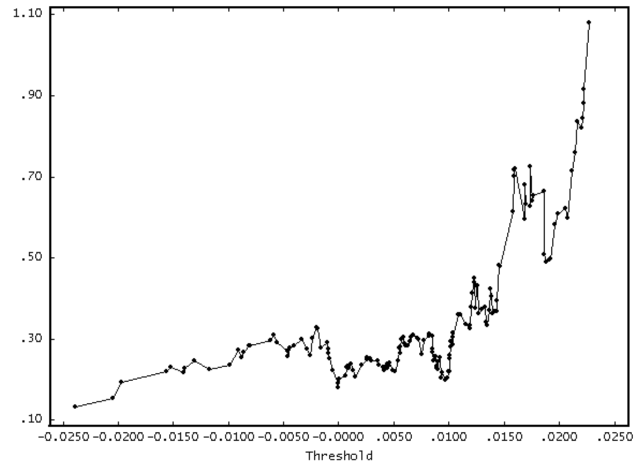
The scatter plots for $\hat{\phi}_1$, $\hat{\phi}_2$, and their t-ratios versus the threshold variable are displayed below.

Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-2}
(GNP series with the Reverse AAR method)

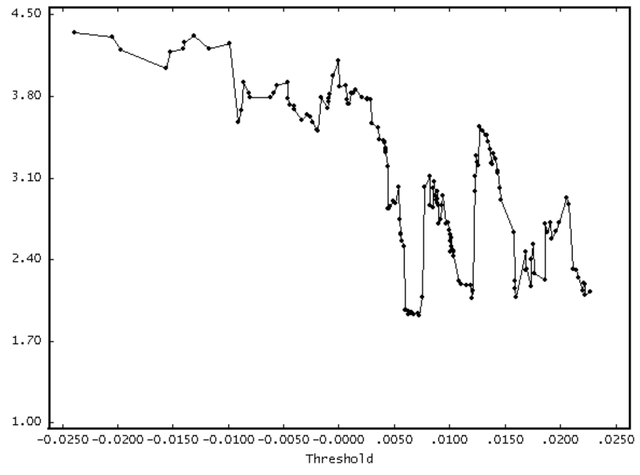
(a) Arranged autoregression estimates of ϕ_1



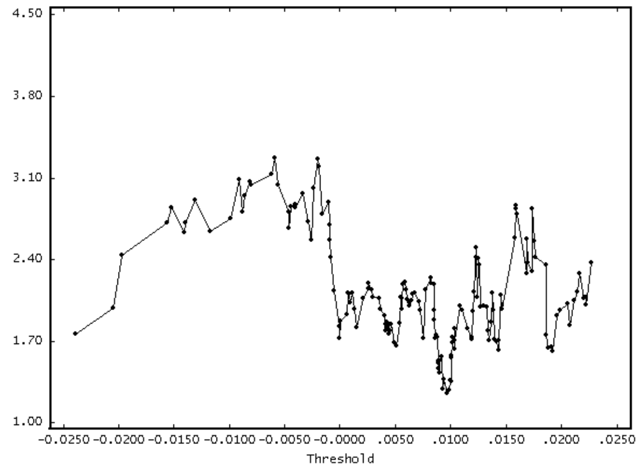
(b) Arranged autoregression estimates of ϕ_2



(c) t-ratios for the estimates of ϕ_1



(d) t-ratios for the estimates of ϕ_2



By examining the dominant coefficient $\hat{\phi}_2$, we find that $\hat{\phi}_2$ and its t-ratios have a major change of direction around $Y_{t-2} = 0.0$ as shown in (b) and (d). Similarly, by examining the coefficient $\hat{\phi}_1$, we find its t-ratios have a major change of direction around $Y_{t-2} = 0.007$. These findings confirm the previous results. Again, the threshold identified based on $\hat{\phi}_2$ is more important than the one based on $\hat{\phi}_1$.

The THMEXPLORE command can also be used to explore potential thresholds using the local arranged regression (LAR) method. To use the LAR method in the THMEXPLORE, a linear regression model must be specified. The model may be in the form of a lagged regression or in the form of a linear regression model that includes explanatory variables.

For comparison purposes, the LAR method will be used to explore the thresholds of the DGNP series. The THMEXPLORE command with a window size of 60 and threshold lag 2 is used. A lagged regression model will be employed using lags one and two of the DGNP series as input variables. The SCA LAG command is used to create lagged variables of the DGNP series stored as YLAG1 and YLAG2. The command is shown below.

```
➔ LAG DGNP.  NEW YLAG1,YLAG2.  LAGS 1, 2.
```

A model is specified in linear regression form using the following TSMODEL command. Here, parameter labels (C, PHI1L, and PHI2L) are specified in the model command.

```
➔ TSMODEL THMODEL.  MODEL IS @
    DGNP=C+(0;PHI1L)YLAG1+(0;PHI2L)YLAG2+NOISE.
```

The THMEXPLORE command is employed using the above model with a moving window size of 60 observations, and threshold lag of 2.

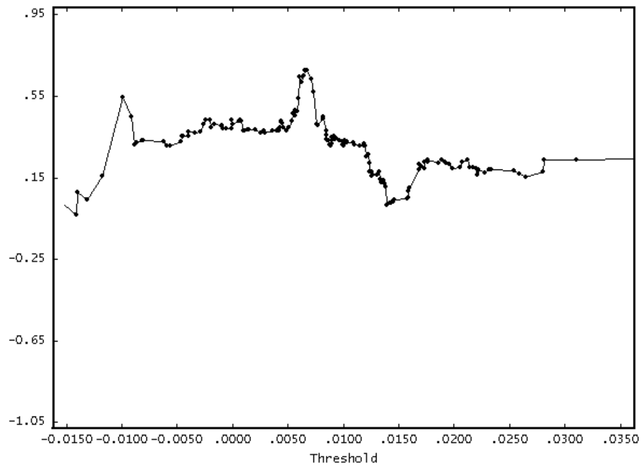
```
➔ THMEXPLORE THMODEL.  THLAG 2.  WINDOWSIZE 60. @
    HOLD THVARIABLE(THVL).
```

```
VARIABLE     THVL  STORES THE SORTED THRESHOLD VARIABLE (WITH   174 VALUES)
VARIABLE        C  STORES PARAMETER ESTIMATES (WITH   174 VALUES)
VARIABLE       _C  STORES THE t-VALUES OF THE ABOVE ESTIMATES
VARIABLE    PHI1L  STORES PARAMETER ESTIMATES (WITH   174 VALUES)
VARIABLE   _PHI1L  STORES THE t-VALUES OF THE ABOVE ESTIMATES
VARIABLE    PHI2L  STORES PARAMETER ESTIMATES (WITH   174 VALUES)
VARIABLE   _PHI2L  STORES THE t-VALUES OF THE ABOVE ESTIMATES
```

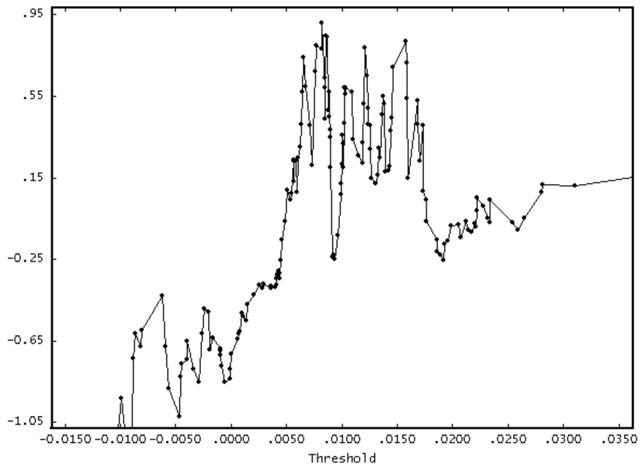
Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-2} are displayed below.

Plots of autoregressive coefficients and their t-ratios versus the threshold variable Y_{t-2} (GNP series with the LAR method)

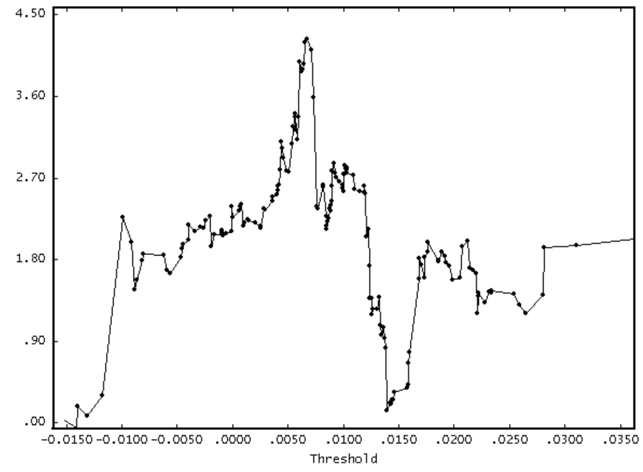
(a) Arranged autoregression estimates of ϕ_1



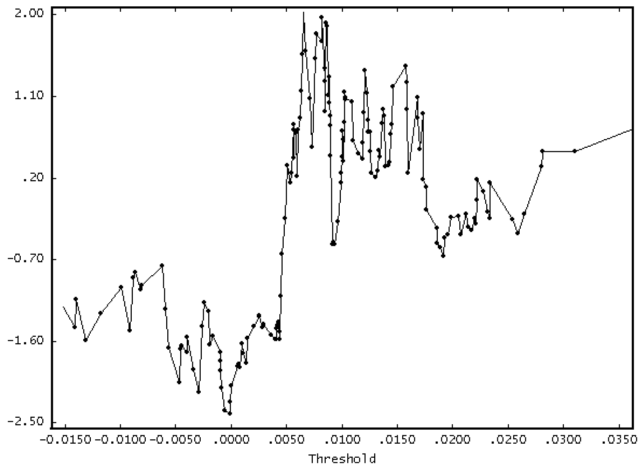
(b) Arranged autoregression estimates of ϕ_2



(c) t-ratios for the estimates of ϕ_1



(d) t-ratios for the estimates of ϕ_2



By examining the dominant coefficient $\hat{\phi}_2$, we find that the estimates of ϕ_2 and their t-ratios have major changes of directions around $Y_{t-2} = 0.0$ and $Y_{t-2} = 0.017$, as shown in (b) and (d). The latter threshold ($Y_{t-2} = 0.017$) should be ignored since the t-ratios before and after this threshold are all insignificant. By examining the coefficient $\hat{\phi}_1$, we find the estimates of ϕ_1 and their t-ratios have a major change of direction around $Y_{t-2} = 0.007$. The findings here are also consistent with the previous results.

Now that the appropriate threshold lag and suitable threshold values have been identified, the REGIME command is used to construct the threshold indicator variables. Here, the threshold lag 2 is specified using the THLAG subcommand and the threshold value 0 is specified using the

THRESHOLD subcommand. The threshold indicator variables are stored as REGIME1 and REGIME2.

➔ REGIME DGNP. THLAG 2. THRESHOLD 0. INDICATORS REGIME1, REGIME2.

Using the regime indicators generated above, the following two-regime TAR(2) is specified using the TSMODEL command and estimated using the TARESTIM command as shown below.

Regime 1: $Y_{t-12} <= 0.0$

➔ TSMODEL MODEL1. MODEL IS (1,2)DGNP=CNST1+NOISE.
➔ TARESTIM MODEL1. REGIME REGIME1.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL1
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

  DGNP     RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------

 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT              ERROR  VALUE
  1  CNST1             CNST      1      0     NONE     -.0039     .0033  -1.18
  2           DGNP      AR       1      1     NONE      .4362     .1754   2.49
  3           DGNP      AR       1      2     NONE     -.7873     .3335  -2.36

TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .           176
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .           174
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .  0.199273E-01
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .  0.120398E-01
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .            37
```

Regime 2: $Y_{t-12} > 0.0$

➔ TSMODEL MODEL2. MODEL IS (1,2)DGNP=CNST2+NOISE.
➔ TARESTIM MODEL2. REGIME REGIME2.

```
SUMMARY FOR UNIVARIATE TIME SERIES MODEL --  MODEL2
-----------------------------------------------------------------------
VARIABLE   TYPE OF    ORIGINAL     DIFFERENCING
          VARIABLE   OR CENTERED

  DGNP     RANDOM     ORIGINAL     NONE
-----------------------------------------------------------------------
 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                 TRAINT              ERROR  VALUE

  1  CNST2             CNST      1      0     NONE      .0038     .0014   2.71
  2           DGNP      AR       1      1     NONE      .3111     .0792   3.93
  3           DGNP      AR       1      2     NONE      .2014     .1061   1.90
```

```
TOTAL NUMBER OF OBSERVATIONS. . . . . . . . . . . . . .            176
EFFECTIVE NUMBER OF OBSERVATIONS. . . . . . . . . . . .            174
RESIDUAL STANDARD ERROR (WITHOUT ANY ADJUSTMENT). . . .   0.988576E-02
RESIDUAL STANDARD ERROR (WITH WEIGHT INFORMATION) . . .   0.869640E-02
NUMBER OF OBS. USED IN COMPUTING THE ABOVE RSE. . . . .            137
```

The above TAR model can be further refined. Tiao and Tsay (1994) employ Y_{t-1} as an additional threshold variable and develop a four-regime TAR model. To simplify our discussion on the issues related to threshold value identification, we focused on the two regime TAR model developed above.

In the above example using the DGNP series, the TARTEST command was employed to test the nonlinearity of DGNP based on lags of the series itself (i.e., traditional TAR model). However, if the underlying model is of general linear regression form, the THMTEST command can be used instead of TARTEST. Once the threshold lag is determined, the THMEXPLORE command can then be used to find the potential thresholds.

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APPENDIX A

SCA COMMAND LIST GROUPED BY FOCUS AND EDITION

This Appendix provides information on the capabilities available in Release 8 of the SCA Statistical System for personal computers. Capabilities are provided in chart form, blocked into the following categories:

- Univariate time series analysis and forecasting
- Automatic univariate time series modeling and outlier analysis
- Multivariate time series modeling and forecasting
- Seasonal adjustment
- Transformation and forecasting
- Nonlinear time series modeling and forecasting
- General statistical analysis
- Descriptive statistics
- Date handling
- Data editing
- Input and output
- Macro procedures
- Utilities
- Applets
- Graphics

Capabilities are listed in alphabetical order by SCA command names within each category. Specific information regarding a capability (command) presented in this appendix include:

- The command name or system function
- A short description for the command
- The SCA Edition(s) that include the command
- The document reference for the command

For convenience, the user guides that document the SCA commands are designated by letter abbreviations listed below:

(A) SCA Reference Manual for Fundamental Capabilities
(B) SCA Reference Manual for General Statistical Analysis
(C) Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 1
(D) Forecasting and Time Series Analysis Using the SCA Statistical System, Volume 2
(E) GARCH Modeling using SCAB34S-GARCH and SCA WorkBench
(F) New and Enhanced Capabilities in Release 8 of the SCA Statistical System (*this document*)
(G) On-line Syntax Help (*Accessible through SCA WorkBench*)

Univariate Time Series Analysis and Forecasting

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
ACF	C	x	x	x	x	Autocorrelation Function
AGGREGATE	C	x	x	x	x	Simple Temporal Aggregation
CCF	C	x	x	x	x	Cross Correlation Function
CCM	C,D	x	x	x	x	Cross Correlation Matrix
CORNER	C	x	x	x	x	Corner Table
CSPECTRA	G	x	x	x	x	Spectral Analysis
EACF	C	x	x	x	x	Extended Autocorrelation Function
ESTIM	C	x	x	x	x	Estimate a time series model
FFILTER	G	x	x	x	x	Band-pass / Band-fail Filter
FILTER	C	x	x	x	x	Filter a time series based on model
FORECAST	C	x	x	x	x	Forecast a time series
GFORECST	C	x	x	x	x	General Exponential Smoothing
IACF	G	x	x	x	x	Inverse Autocorrelation Function
IDENTIFY	C	x	x	x	x	ACF & PACF
OUTLIER	C	x	x	x	x	Outlier detection and identification
PACF	C	x	x	x	x	Partial Autocorrelation Function
PSPECTRA	G	x	x	x	x	Spectral Analysis
SIMULATION	C	x	x	x	x	Simulate a time series based on model
RSFILTER	F	x	x	x	x	Seasonal time series model identification
TSMODEL	C	x	x	x	x	Time series model specification
UROOT	F	x	x	x	x	Unit Root tests
WEIGHT	C	x	x	x	x	Pi-weights and Psi-weights

Automatic Univariate Time Series Analysis and Forecasting

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
IARIMA	D	--	x	x	x	Automatic ARIMA Modeling
IESTIM	D	--	x	x	x	Automatic ARIMA & Transfer Function
OESTIM	C	--	x	x	x	Joint Outlier Estimation and Adjustment
OFILTER	C	--	x	x	x	Outlier Filtering of a Time Series
OFORECST	C	--	x	x	x	Outlier Adjusted Forecasting
OUTLIER	C	--	x	x	x	Outlier detection and identification
RSFILTER	F	x	x	x	x	Seasonal time series model identification

Multivariate Time Series Analysis and Forecasting

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
CAUSALTEST	F	--	--	x	--	Causality tests
CANONICAL	G	--	--	x	--	Canonical Analysis
CCM	D	x	x	x	x	Cross Correlaton Matrix
ECCM	D	--	--	x	--	Extended Cross Correlation Matrix
MESTIM	D	--	--	x	--	Vector ARMA Model Estimation
IMESTIM	D	--	--	x	--	Automatic Vector ARMA Estimation
MFORECAST	D	--	--	x	--	Vector ARMA Forecasting
MIDEN	D	--	--	x	--	Vector ARMA Model Identification
MSIMULATE	D	--	--	x	--	Vector ARMA Model Simulation
MTSMODEL	D	--	--	x	--	Vector ARMA Model Specification
SCAN	G	--	--	x	--	Vector ARMA Model Identification
SESTIM	D	--	--	x	--	STF Model Estimation
SFORECAST	D	--	--	x	--	STF Model Forecasting
SSIMULATE	D	--	--	x	--	STF Model Simulation
STEPAR	D	--	--	x	--	Step-wise Autoregressive Fit
STFMODEL	D	--	--	x	--	STF Model Specification

Seasonal Adjustment

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
XCD	D	--	--	x	--	Model-based seasonal adjustment

Transformation and Forecasting

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
RETRANSFORM	F	--	x	x	x	Retransform forecasts (unbias)
TSEARCH	F	--	--	--	x	Lambda value search for transformation

Nonlinear Time Series Modeling and Forecasting

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
GARCH	E	--	--	--	x	GARCH modeling and analysis environment
NLTEST	F	--	--	--	x	Nonlinearity testing using LM ARCH test
REGIME	F	--	--	--	x	Generate binary regime indicator variables
SPLINES	G	*	*	*	*	MARSplines, GAM, ACE, other
TARESTIM	F	--	--	--	x	Threshold AR estimation
TARFORECAST	F	--	--	--	x	Threshold AR forecast
TARTEST	F	--	--	--	x	Threshold AR F-Test and threshold lag search
TARXTEST	F	--	--	--	x	Threshold search using external variable
THMEXPLORE	F	--	--	--	x	Threshold search using arranged regression
THMTEST	F	--	--	--	x	Threshold AR F-Test and threshold lag search
TSEARCH	F	--	--	--	x	Lambda value search for transformation
TVPEXPLORE	F	--	--	--	x	Time-varying parameter analysis
WESTIM	F	--	--	--	x	Weighted time series model estimation
WFORECAST	F	--	--	--	x	Weighted forecasting

(*) The SPLINES capability requires the SCAB34S SPLINESproduct.

General Statistical Analysis

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
CORRELATION	B	x	x	x	x	Display correlation between two variables
CROSSTAB	B	x	x	x	x	Cross tabulation
NONPARAMETRIC	B	x	x	x	x	Non-parametric statistics
NWAY	B	x	x	x	x	N-way analysis of variance
OWAY	B	x	x	x	x	One-way analysis of variance
PTRAN	B	x	x	x	x	Parameter estimation with Box-Cox transform
RANK	B	x	x	x	x	Convert data into a ranked variable
REGRESS	B	x	x	x	x	Estimate a regression model
TABLE	B	x	x	x	x	Display sample mean and STD of data groups
TTEST	B	x	x	x	x	Two-sample t-test
TWAY	B	x	x	x	x	Two-way analysis of variance

Date Handling

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
DAGGREGATE	F	--	x	x	x	Temporal aggregation using date indices
DATEBUILD	F	--	x	x	x	Generate date variable
DAYS	C	--	x	x	x	Compute number of mondays, etc. in month
DMATRIX	G	x	x	x	x	Generate a design matrix from factor variable
DOWEEK	F	--	x	x	x	Day of week associated with date
DVECTOR	F	--	x	x	x	Generate dummy variables from factors
EASTER	C	--	x	x	x	Generate monthly weights related to Easter

Data Editing

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
AGGREGATE	A,C	x	x	x	x	Temporal aggregation using observation index
AUGMENT	A	x	x	x	x	Join variables into a matrix
CHANGE	A	x	x	x	x	Change the values of a variable
DIFFERENCE	A,C	x	x	x	x	Difference a time series
DMATRIX	G	x	x	x	x	Generate a design matrix from factor variable
GENERATE	A	x	x	x	x	Generate a variable using specified sequence
JOIN	A	x	x	x	x	Join variables together as a new variable
LAG	A,C	x	x	x	x	Lag a time series
OMIT	A	x	x	x	x	Omit values or time span in a variables
PATCH	A,C	x	x	x	x	Patch or recode missing values
PERCENT	A,C	x	x	x	x	Generate a percent change variable
PICK	A	x	x	x	x	Pick off columns from a matrix
RANK	A	x	x	x	x	Convert data into a ranked variable
RECODE	A	x	x	x	x	Recode values in a variables to a new value
SELECT	A	x	x	x	x	Select obs. of a variable by value or span
SIMULATE	A	x	x	x	x	Simulate a data with a specified distribution
SORT	A	x	x	x	x	Sort the values of a variable

Descriptive Statistics

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
DESCRIBE	B	x	x	x	x	Display descriptive statistics of a variable
DPLOT	G	x	x	x	x	Dispersion plot (character plot)
GRAPH	G	x	x	x	x	Create graph in SCAGRAF Applet (graphic plot)
HGRAPH	G	--	--	--	--	Create graph in SCAB34S Applet (graphic plot)
HISTOGRAM	B	x	x	x	x	Histogram plot (character plot)
LABEL	A	x	x	x	x	Assign one character label to variable name
MPLOT	A	x	x	x	x	Multiple scatter plot (character plot)

MTPLOT	A	x	x	x	x	Multiple time series plot (character plot)
MTSPLOT	A	x	x	x	x	Multiple time series plot (character plot)
PARETO	A	x	x	x	x	Pareto plot (character plot)
PLOT	A	x	x	x	x	Scatter plot (character plot)
PPLOT	A	x	x	x	x	Probability plot (character plot)
SHEWHART	A	x	x	x	x	Shewhart plot (character plot)
TPLOT	A	x	x	x	x	Time series plot (character plot)
TSPLOT	A	x	x	x	x	Time series plot (character plot)

Input and Output

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
ASSIGN	A	x	x	x	x	Associate a file with a unit number
BINPUT	A	x	x	x	x	Input dataset using SCA binary format
BSAVE	A	x	x	x	x	Save dataset using SCA binary format
DISPLAY	A	x	x	x	x	Display descriptive text in SCA macro
FINPUT	A	x	x	x	x	Input dataset using SCA meta data format
FREE	A	x	x	x	x	Release a unit number associated with a file
FSAVE	A	x	x	x	x	Save dataset using SCA meta data format
INPUT	A	x	x	x	x	Input data from a flat text file
PRINT	A	x	x	x	x	Print variables
REWIND	A	x	x	x	x	Rewind a unit number associated with a file
SAVE	A	x	x	x	x	Save data in a flat text file format

Macro Procedures

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
CALL	A	x	x	x	x	Call a SCA macro procedure script
PARAMETERS	A	x	x	x	x	Specify symbolic parameters
RETURN	A	x	x	x	x	Return from macro procedure

Utilities

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
NEWPAGE	A	x	x	x	x	Force start of new page in SCA output
PROFILE	A	x	x	x	x	Control key settings of SCA System session
RESTART	A	x	x	x	x	Clears the SCA workspace
STOP	A	x	x	x	x	Exit the SCA System session
TIME	A	x	x	x	x	Print date and time
WORKSPACE	A	x	x	x	x	Provides various SCA Workspace utilities

Applets

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
RUN	G	x	x	x	x	Execute an external program
RUNW	G	x	x	x	x	Execute an external program and wait for finish
BUILD	G	x	x	x	x	Write text to a file in free format (control file)
ENDBUILD	G	x	x	x	x	End writing text to a file
COPY	G	x	x	x	x	Copy text from external file into the SCA output
XLSREAD	G	x	x	x	x	Build SCA data macro from Excel file
XLSWRITE	G	x	x	x	x	Write data from a file into Excel

Graphics

Command	User Guide	Educational	Practitioner	Professional A	Professional B	Description
HGRAPH	G	*	*	*	*	Create graph in SCAB34S Applet (graphic plot)
SCAGRAF	G	x	x	x	x	Create graph in SCAGRAF Applet (graphic plot)

(*) The HGRAPH command requires the SCAB34S product.

APPENDIX B

SCA COMMAND SYNTAX LISTING

This appendix B provides a listing of the syntax for new SCA commands included in Release 8 of the SCA Statistical System. The commands are presented in alphabetical order.

CAUSALTEST

The CAUSALTEST command is used to examine the causal relationships between two time series.

SYNTAX in terms of subcommands and associated argument type(s)

```
+-----------------------------------------------------------------+
|                                                                 |
|  CAUSALTEST MODEL model-name.                                   |
|             PROCEDURE IS w.                                @   |
|             ALPHA IS r.                                        |
|                                                                 |
|  ** Complete list of available subcommands:                     |
|                                                                 |
|  CAUSALTEST MODEL model-name.                              @   |
|             PROCEDURE IS w.                                @   |
|             ALPHA IS r.                                    @   |
|             STOP-CRITERIA ARE MAXIT(i), LIKELIHOOD(r).     @   |
|             SPAN IS i1, i2.                                @   |
|             OUTPUT IS LEVEL(w), PRINT(w1,w2,---),          @   |
|                       NOPRINT(w1,w2,---).                  @   |
|                                                                 |
|     Required subcommand:  MODEL                                 |
|                                                                 |
|     Legend:  v -- variable namel   i -- integer value;          |
|              w -- keyword          r -- real value              |
+-----------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the name (label) of the vector ARMA model used for causality testing. The name must be a vector ARMA model specified in a previous TSMODEL command.

PROCEDURE subcommand

The PROCEDURE subcommand is used to specify the approach employed in causality testing. The keyword, w, may be BACKWARD for the backward procedure, FORWARD for the forward procedure, or BOTH for both backward and forward procedures. The default is BOTH.

ALPHA subcommand

The ALPHA subcommand is used to specify the significant level employed in conducting all pair-wise tests. The default is 0.05 (i.e., 5%).

STOP subcommand

The STOP subcommand is used to specify the stopping criterion for nonlinear estimation. The argument, i, for the keyword MAXIT specifies the maximum number of iterations (default is i=10), and the argument, r, for the keyword LIKELIHOOD specifies the value of the relative convergence criterion on the likelihood function (default is r=0.001). Estimation iterations will be terminated when the relative change in the value of likelihood function between two successive iterations is less than or equal to the convergence criterion, or if the maximum number of iterations is exceeded.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which the data will be analyzed. The default is the maximum span available for the series which is the span of the shortest series if all series are not of equal length.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed for computed statistics. Control is achieved in a two stage procedure. First a basic LEVEL of output (default NORMAL) is specified. Output may then be increased from this level by use of PRINT (NOPRINT).

The keywords for LEVEL and output printed are:

BRIEF:	likelihood values and their related statistics only
NORMAL:	same as BRIEF
DETAILED:	CORR

where the reserved words (and keywords for PRINT, NOPRINT) on the right denote:

CORR:	the correlation matrix for the parameter estimates. It also activates the display of detailed information in causality testing.

DAGGREGATE

The DAGGREGATE command is used to generate a new time series through the temporal aggregation of a specified time series given a companion date index variable. The generated series will be aggregated into user specified time intervals such as year, quarter, month, week, or day. Aggregation method may be specified as the sum, mean, first, last, high or low of the data values in each date period.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|   DAGGREGATE OLD v1,v2,--- .                           @  |
|              NEW v1,v2,--- .                           @  |
|              DATE IS v.                                @  |
|              METHOD IS w1(w2).                            |
|                                                                |
|   ** Complete list of available subcommands:                   |
|                                                                |
|   DAGGREGATE OLD v1,v2,--- .                           @  |
|              NEW v1,v2,--- .                           @  |
|              DATE IS v.                                @  |
|              METHOD IS w1(w2).                         @  |
|              WBEGIN IS i.                              @  |
|              COMPRESS/NO COMPRESS.                     @  |
|              HOLD DATE(v),NOBS(v1,v2,---),MEAN(v1,v2,---),@  |
|                   STDERR(v1,v2,---).                      |
|                                                                |
|      Required subcommands: OLD, NEW, DATE, METHOD              |
|                                                                |
|      Legend:  v -- variable name;    i -- integer value;       |
|               w -- keyword                                     |
+----------------------------------------------------------------+
```

Subcommand descriptions

OLD subcommand

The OLD subcommand is used to specify the name(s) of time series variable(s) from which the aggregated time series will be derived. It is a required subcommand.

NEW subcommand

The NEW subcommand is used to specify the name(s) of variable(s) to store the aggregated time series. The default are the names specified in the OLD subcommand. It is a required subcommand.

DATE subcommand

The DATE subcommand is used to specify the name of the date index variable associated with the original series to be aggregated. The date must be a 8-digit number representing YYYYMMDD where YYYY is year, MM is month, and DD is day. It is a required subcommand.

METHOD subcommand

The METHOD subcommand is used to specify the aggregation grouping and the summarization or aggregation method. The aggregation grouping keyword (w1) can be specified as YEAR, QUARTER, MONTH, WEEK, or DAY. The aggregation method keyword (w2) can be specified as SUM, MEAN, FIRST, LAST, HIGH, or LOW. It is a required subcommand.

WBEGIN subcommand

The WBEGIN subcommand is used to specify the first day of the week which is only applicable to weekly aggregation. For example, the weekly data aggregation will begin on Tuesday if 2 is specified.

COMPRESS subcommand

The COMPRESS subcommand is used to specify that the aggregated series shall be stored in compressed form, i.e., aggregated values are not repeated so that the total number of observations are less than that of the original series. This is the typical case. If NO COMPRESS is specified, then the aggregated series will have the same length as the original series. The default is COMPRESS.

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those information desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the above information will be retained after the command is used. The values that may be retained are:

DATE:	the new dates associated with the aggregated series. For aggregation in WEEK, the values stored in the DATE variable are 10 digit values where the first 8 digits represent the date for each aggregated value, and the last 2 digits represent the week for the aggregated value.
NOBS:	the number of observations in the each aggregated period. This information is useful to evaluate and adjust partially aggregated periods
MEAN:	the mean of the observations in the aggregated period.
STDERR:	the sample standard deviation of the observations in the aggregated period

DATEBUILD

The DATEBUILD command is used to generate a sequence of dates given a beginning period and an ending period. If the ending period is not well defined, the number of dates to create may be specified directly using the NOBS subcommand.

SYNTAX in terms of subcommands and associated argument type(s)

```
+---------------------------------------------------------------+
|                                                               |
|      DATEBUILD VARIABLE IS v.                         @       |
|                BEGIN i1,i2,i3.                        @       |
|                END i1,i2,i3.                          @       |
|                NOBS IS i.                                     |
|                                                               |
|   ** Complete list of available subcommands:                  |
|                                                               |
|      DATEBUILD VARIABLES IS v.                        @       |
|                QUARTERLY.                             @       |
|                BEGIN i1,i2,i3.                        @       |
|                END i1,i2,i3.                          @       |
|                NOBS IS i.                             @       |
|                OMIT DOWEEK(---),DAY(---),MONTH(---),  @       |
|                     QUARTER(---),YEAR(---).           @       |
|                HOLD YEAR(v),QUARTER(v),MONTH(v),DAY(v), @     |
|                        DOWEEK(v).                             |
|                                                               |
|      Required subcommands:  VARIABLES, BEGIN, END (or NOBS)   |
|                                                               |
|      Legend:  v -- variable name                              |
|                                                               |
+---------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the name of the variable that will hold the newly generated date values.

QUARTERLY subcommand

The QUARTERLY subcommand is used to specify that the date variable shall be organized as quarterly dates.

BEGIN subcommand

The BEGIN subcommand is used to specify the beginning period for the generated date variable. The first parameter value i1 specifies the beginning year as a 4-digit integer, the second parameter value i2 specifies the beginning month or quarter (if the QUARTERLY subcommand is specified), and the third parameter value i3 specifies the beginning day. If only the year is needed, it is not necessary to provide values for i2 and i3. Similarly, if only year and month is needed, it is not necessary to provide a value for i3.

END subcommand

The END subcommand is used to specify the ending period for the new date variable. The first parameter value i1 specifies the beginning year as a 4-digit integer, the second parameter value i2 specifies the beginning month or quarter (if the QUARTERLY subcommand is specified), and the third parameter value i3 specifies the beginning day. If only the year is needed, it is not necessary to provide values for i2 and i3. Similarly, if only year and month is needed, it is not necessary to provide a value for i3. The NOBS subcommand can be used instead of the END subcommand if the ending date is not well defined.

NOBS subcommand

The NOBS subcommand is used to specify the number of dates that are to be generated starting from the specified beginning date period.

OMIT subcommand

The OMIT subcommand is used to specify if certain date periods are to be skipped when building the date variable. For example, weekend dates can be omitted in daily date creation by specifying OMIT DOWEEK(6,7).

HOLD subcommand

The HOLD subcommand is used to store the components of the generate date variable into separate variables. The component values that may be retained are:

YEAR:	the years associated with the dates generated
QUARTER:	the quarters associated with the dates generated
MONTH:	the months associated with the dates generated
DAY:	the days associated with the dates generated
DOWEEK:	the day of week associated with the dates generated where Monday=1, Tuesday=2, ..., Sunday=7.

DOWEEK

The DOWEEK command is used to evaluate a double-precision date variable and determine the day of week associated with that date. The date variable must be a double precision variable of the form YYYYMMDD where YYYY represents the 4-digit year, MM represents the two-digit month, and DD represents the two-digit day. The command returns 1 for Monday, 2 for Tuesday, 3 for Wednesday, 4 for Thursday, 5 for Friday, 6 for Saturday, and 7 for Sunday.

SYNTAX in terms of subcommands and associated argument type(s)

```
+--------------------------------------------------------------+
|                                                              |
|      DOWEEK  VARIABLE IS v.                           @      |
|              DAYOFWEEK IS v.                                 |
|                                                              |
|   ** Complete list of available subcommands:                 |
|                                                              |
|      DOWEEK  VARIABLE IS v.                           @      |
|              DAYOFWEEK IS v.                          @      |
|              DATEPARTS IN v1,v2,v3.                          |
|                                                              |
|      Required subcommands:  VARIABLES and DAYOFWEEK          |
|                                                              |
|      Legend:  v -- variable name                             |
|                                                              |
+--------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand
The VARIABLE subcommand is used to specify the name of the variable whose values represent dates in the form YYYYMMDD. It is a required subcommand.

DAYOFWEEK subcommand
The DAYOFWEEK subcommand is used to specify the variable (label) used to hold the associated day of week values. It is a required subcommand.

DATEPARTS subcommand
The DATEPARTS subcommand stores the date information into separate variables where years will be saved to first variable name specified, months will be saved to the second variable name, and days will be saved to the third variable name specified.

DVECTOR

The DVECTOR command is used to construct a set of dummy variables (design matrix) from the values of factor variables and stores the dummy variables as vector variables. The resultant dummy variables can then be used in subsequent time series modeling and forecasting. Its functionality is similar to that of the DMATRIX command, but the resultant dummy variables are in vector form (as opposed to matrix form) and are more convenient to use in time series modeling and forecasting.

SYNTAX in terms of subcommands and associated argument type(s)

```
+--------------------------------------------------------------+
|                                                              |
|      DVECTOR  VARIABLES ARE v1, v2, ---.            @        |
|               MAIN-EFFECTS ARE v1, v2, ---.                  |
|                                                              |
|   ** Complete list of available subcommands:                 |
|                                                              |
|      DVECTOR  VARIABLES ARE v1, v2, ---.            @        |
|               MAIN-EFFECTS ARE v1, v2, ---.         @        |
|               TYPE IS DEVIATION/T11/T10/T01.        @        |
|               INTERACTIONS ARE v1(...), v2(...), ---.        |
|                                                              |
|      Required subcommands:  VARIABLES and MAIN-EFFECTS       |
|                                                              |
|      Legend:  v -- variable name                             |
|                                                              |
+--------------------------------------------------------------+
```

Subcommand descriptions

VARIABLES subcommand

The VARIABLES subcommand is used to specify the names of the variables whose values are used to define the resultant design matrices. The variables must be categorical and specified as numerical values.

MAIN-EFFECTS subcommand

The MAIN-EFFECTS subcommand is used to specify the variables (labels) for holding the resultant dummy variables. One set of dummy variables is constructed for each of the variables specified in the VARIABLES subcommand. Each dummy variable will have as many rows as the original variable. The number of dummy variables generated for each variable may be equal to or one less than the number of categories for the associated variable depending on the choice of the TYPE of design matrix formulation. The number of labels specified in the MAIN-EFFECTS subcommand must be the same as the number of variables specified in the VARIABLES subcommand. The labels specified here are used as the root word in storing the individual dummy variable vectors for the associated main effect. Since SCA has a limit of 8 characters for variable labels, it is important to use short labels of 6 characters or less. For example, if there are 4 categories in a variable and a root word label is specified as QA, the dummy variable vector labels associated with that variable will be assigned as QA1, QA2, and QA3 (and QA4 if TYPE is T11, see below).

TYPE subcommand

The TYPE subcommand is used to specify the formulation of the design matrix. The keyword DEVIATION will produce a set of dummy variables such that its use in a regression or time series model will provide the estimates of the main-effects as deviations from the overall mean. In this case, there will be (m-1) dummy variables created for the variable where m is the number of categories. The keyword T11 will generate a dummy variable for each category in the variable. In this case, there will be m dummy variables created for the variable. The keyword T10 is similar to T11 except the dummy variable for the last category will not be created. The keyword T01 is also similar to T11 except the dummy variable for the first category will not be created. Both T10 and T01 create (m-1) dummy variables for each associated variable. The default is DEVIATION.

INTERACTION subcommand

The INTERACTION subcommand is used to specify the construction of design matrices for interactions. The specification of each interaction is given by v (v1, v2, ---) where 'v' is the variable name (label) used to store the resultant design matrix for an interaction; and "v1, v2, ---" specifies the terms whose cross-products form the interaction, and have been specified previously in the MAIN-EFFECTS subcommand. Note that not all variables specified in the VARIABLES subcommand need be used in this subcommand and more than one interaction matrix can be constructed.

IARIMA

The IARIMA command is used to automatically identify an appropriate ARIMA model for a seasonal or non-seasonal time series. It also estimates the parameters of the identified model.

SYNTAX in terms of subcommands and associated argument type(s)

```
+--------------------------------------------------------------+
|                                                              |
|     IARIMA  VARIABLE IS v.                        @          |
|             SEASONALITY is i.                     @          |
|             SPAN IS i1, i2.                       @          |
|             DFORDERS ARE  i1, i2, --- .           @          |
|             NODFORDERS i1, i2, --- .                         |
|                                                              |
|   ** Complete list of available subcommands:                 |
|                                                              |
|     IARIMA  VARIABLE IS v.                        @          |
|             NAME IS model-name                    @          |
|             SEASONALITY is i. (PERIOD IS i.)      @          |
|             SPAN IS i1, i2.                       @          |
|             DFORDERS ARE  i1, i2, --- .           @          |
|             NODFORDERS i1, i2, --- .              @          |
|             CRITERIA r.                           @          |
|             DELETE-CONSTANT/NO DELETE.            @          |
|             REPLACE IN model-name.                @          |
|             COMPONENT-SERIES ARE v1,v2,v3.        @          |
|             HOLD RESIDUALS(v),FITTED(v),VARIANCE(v).         |
|                                                              |
|      Required subcommand:  VARIABLE                          |
|                                                              |
|      Legend:  v -- variable name;    i -- integer value;     |
|               w -- keyword                                   |
+--------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the name of the series for which an ARIMA model will be identified and estimated. It is a required subcommand.

NAME subcommand

The NAME subcommand is used to specify a name (label) for the model identified by the IARIMA command. When it is not specified, the default name UTSMODEL is used internally.

SEASONALITY subcommand

The SEASONALITY subcommand is used to specify the potential seasonality the series may possess. The seasonality is 4 for quarterly data, 12 for monthly data, and so on. If seasonality is specified but in fact the series is non-seasonal, an appropriate non-seasonal model will still be obtained (assuming that the series is medium to long in length). If a series is seasonal but no seasonality is specified, the identified model will not be appropriate (it is signified by the display of significant sample autocorrelations of residuals). Hence the SEASONALITY is required if the series is seasonal, and optional if the series is non-seasonal. It is safe to specify the potential seasonality if the series is medium to long in length.

PERIOD subcommand

The PERIOD subcommand serves the same purpose as the SEASONALITY subcommand. Either PERIOD or SEASONALITY may be used to specify the periodicity or seasonality of a time series, but not both.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which the data are used to identify and estimate an ARIMA model. The default is the maximum span available for the series.

DFORDER subcommand

The DFORDER subcommand is used to specify the differencing order(s) that must be included in the final ARIMA model. In addition to the specified differencing(s), other differencing orders may be included by the IARIMA command in the final ARIMA model if they are found to be necessary. By default, the differencing orders for the ARIMA model of a time series are automatically determined by the IARIMA command. User imposed specification of differencing order(s) may be particularly useful when a time series is short.

NODFORDERS subcommand

The NODFORDERS subcommand is used to exclude specific differencing order(s) from being used in the final ARIMA model. By default, the differencing orders for the ARIMA model of a time series are automatically determined by the IARIMA command.

CRITERIA subcommand

The CRITERIA subcommand is used to specify the critical value for determining the statistical significance of parameters retained in the model. The default is 1.96 for determining the statistical significance of model parameters.

DELETE-CONSTANT subcommand

The DELETE-CONSTANT subcommand is used to specify the manner in which the deletion of the constant term in an ARIMA model is handled. By default (which is DELETE-CONSTANT), the constant term is deleted from the model if it is insignificant, and retained in the model if it is significant. However, if NO DELETE is specified, the constant term will be retained in the model no matter if it is significant or not. The default is DELETE-CONSTANT.

REPLACE subcommand

The REPLACE subcommand is used to specify the name of a transfer function model or intervention model for which its ARMA component is to be replaced by the ARIMA model identified by the IARIMA command. The REPLACE subcommand provides a link for the combined use of the IARIMA and IESTIM commands.

COMPONENT subcommand

The COMPONENT subcommand is used to specify the names of variables to store the residual series, R series (nonseasonal), and S series (seasonal). See SCA manual for the definition of R and S series. The S series will not be generated if the SEASONALITY (or PERIOD) subcommand is not specified.

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the above statistics will be retained after the command is used. The values that may be retained are:

RESIDUALS:	the residual series
FITTED:	the one-step-ahead forecasts (fitted values) of the series
VARIANCE:	the variance of the noise

NLTEST

The NLTEST command performs a non-linear testing of a series using a LaGrange multiplier approach proposed in Engle (1982) to test the null hypothesis that errors do not follow an ARCH process. The test performs a regression of the squared residuals on a constant and q lagged values of the squared residuals. From the regression, the LM test statistic is calculated as (N-q)-R**2. An ARCH process is determined to exist if the LM test exceeds the critical value from a chi-square distribution with q degrees of freedom.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|   ** Complete list of available subcommands:                   |
|                                                                |
|      NLTEST  VARIABLE IS v1.                            @      |
|              METHOD IS w.                               @      |
|              ORDERS ARE i1,i2 ---.                      @      |
|              SPAN IS i1,i2.                                    |
|                                                                |
|      Required subcommand:                                      |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;         |
|               w -- keyword;        r -- real value             |
+----------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the series on which a nonlinear test is conducted.

METHOD subcommand

The METHOD subcommand is used to specify the type of nonlinear test to be employed. The valid keywords are LM (LaGrange Multiplier).

ORDERS subcommand

The ORDERS subcommand is used to specify the lag order(s) to be tested.

SPAN subcommand

The SPAN subcommand is used to specify the span of the series to be considered. The default is all available data.

REGIME

The REGIME command is used to generate binary indicator variables based on the thresholds specified for a given time series.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|      REGIME  VARIABLE IS v.                           @    |
|              THLAG IS i.                              @    |
|              THRESHOLDS ARE r1, r2, ---.              @    |
|              INDICATORS ARE v1, v2, v3, ---.               |
|                                                                |
|   ** Complete list of available subcommands:                |
|                                                                |
|      REGIME  VARIABLE IS v1.                          @    |
|              THLAG IS i.                              @    |
|              THRESHOLDS ARE r1, r2, ---.              @    |
|              INDICATORS ARE v1, v2, v3, ---.          @    |
|              FINDICATORS ARE v1, v2, v3,---.          @    |
|              EXTEND. / NO EXTEND.                          |
|                                                                |
|      Required subcommand:   VARIABLE, THRESHOLDS, INDICATORS  |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;        |
|               w -- keyword;        r -- real value            |
+----------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the time series whose values will be evaluated in generating the regime indicator variables (i.e., the regime variables).

THLAG subcommand

The THLAG subcommand is used to specify the threshold lag that is used to generate the regime variables. The default is 1.

THRESHOLDS subcommand

The THRESHOLDS subcommand is used to specify the cut-off values that define the threshold intervals. More than one threshold value may be specified but must be specified in ascending order.

INDICATORS subcommand

The INDICATORS subcommand is used to specify the names of the variables that will hold the regime indicator variables. The number of regime indicator variables generated is always one greater than the number of threshold values specified.

If two threshold values are specified, three regime indicator variables will be generated. The first regime variable will mark all observations that are less than the first threshold value. The second regime indicator will mark all observations that are greater than or equal to the first threshold value but less than the second threshold value. The third regime indicator will mark all observations that are greater than the second threshold value.

FINDICATORS subcommand

The FINDICATORS subcommand is used to specify the names of the regime indicator variables that hold the indicator information that is extended past the length of the time series. These regime indicator variables can then be used for forecasting a TAR model up to threshold lag value specified. The threshold lag must be greater than 0 when using the FINDICATOR subcommand.

EXTEND subcommand

The EXTEND subcommand is used to extend the regime indicator variables by the value specified in the THLAG subcommand. The default is NO EXTEND.

RETRANSFORM

The RETRANSFORM command is used to retransform forecasts back into original scale when a time series received a power transformation.

SYNTAX in terms of subcommands and associated argument type(s)

```
+-----------------------------------------------------------------+
|                                                                 |
|     RETRANSFORM OLD-VARIABLES ARE v1, v2, v3.  @                |
|                 FORM IS v1, v2, v3, v4.        @                |
|                 NEW-VARIABLES ARE v1, v2, v3.  @                |
|                 CRITICAL-VALUE r.              @                |
|                 METHOD IS w.                                    |
|                                                                 |
|   ** Complete list of available subcommands:                    |
|                                                                 |
|     RETRANSFORM OLD-VARIABLES ARE v1, v2.      @                |
|                 FORM IS v1, v2, v3, v4.        @                |
|                 NEW-VARIABLES ARE v1, v2, v3.  @                |
|                 CRITICAL-VALUE r.              @                |
|                 METHOD IS w.                   @                |
|                 OUTPUT LEVEL(W),PRINT(...),NOPRINT(...)         |
|                                                                 |
|      Required subcommand:   OLD-VARIABLES, FORM                 |
|                                                                 |
|      Legend:  v -- variable name;  i -- integer value;          |
|               w -- keyword;        r -- real value              |
+-----------------------------------------------------------------+
```

Subcommand descriptions

OLD-VARIABLES subcommand

The OLD-VARIABLES subcommand is used to specify the forecasts and the forecast standard errors which are based on a transformed time series. The argument, v1, specifies the forecast series. The argument, v2, specifies the standard errors of the forecasts. The argument, v3, specifies a vector containing the actual values for the forecast periods, if known. The actual values should be in original scale (i.e., prior to being transformed). The v3 argument must be the same length as v1. If specified, the result table will be expanded to show the forecast errors, as well as the RMSE and MAPE statistics of the forecasts based on the original scale of the series.

FORM subcommand

The FORM subcommand is used to specify the form of retransformation method applied.

The argument, v1, specifies the power (lambda) value associated with the forecasts to be retransformed. This is a required argument.

The argument, v2, is used to specify the type of transformation applied to the transformed forecasts. The v2 argument must pass the value 1 or 2. An explanation of Type 1 and Type 2 transformation is provided in the body of the text. If the argument, v2, is not specified, Type 1 transformation form is assumed.

The optional argument, v3, specifies if a constant was added to the original series before it received a power transformation.

The optional argument, v4, specifies if the transformed forecasts were scaled by the geometric mean of the original series, or by any other arbitrary value.

NEW-VARIABLES subcommand

The NEW-VARIABLES subcommand is used to specify the Retransformed forecasts and upper/lower confidence limits of the Retransformed forecasts. The argument, v1, specifies the Retransformed forecast series. The arguments, v2 and v3, specify the upper limit and lower limit of the retransformed forecasts, respectively.

CRITICAL-VALUE

The CRITICAL-VALUE subcommand is used to specify the critical value for computing the confidence intervals of the Retransformed forecasts. The default is 1.96 (i.e., 95% confidence intervals).

METHOD subcommand

The METHOD subcommand is used to specify the method for transforming the forecasts back into original scale. The method may be specified as STRAIGHT or UNBIASED. The default is UNBIASED.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed for the Retransformed series. Control is achieved in a two stage procedure. First a basic LEVEL of output is specified. Output may then be increased from this level by use of PRINT, or decreased from this level by use of NOPRINT.

The keywords for LEVEL and output printed are:

BRIEF:	no output displayed
NORMAL:	FORECASTS
DETAILED:	FORECASTS

where the reserved words on the right denote:

FORECAST:	the Retransformed forecast

These reserved words are also keywords for PRINT and NOPRINT. The default for LEVEL is NORMAL, or the level specified in the PROFILE command.

RSFILTER

The RSFILTER command is used to identify the differencing orders and generate the R and S series for a seasonal time series after differencing. This command provides more detailed information for the identification of seasonal ARIMA models. The R series contains information to represent the behavior for the nonseasonal part of the ARMA process, and the S series contains information to represent the behavior for the seasonal part of the ARMA process.

SYNTAX in terms of subcommands and associated argument type(s)

```
+---------------------------------------------------------------+
|                                                               |
|    RSFILTER VARIABLE IS v.                        @           |
|             NEW-SERIES ARE v1,v2.                 @           |
|             SEASONALITY is i. (PERIOD IS i.)                  |
|                                                               |
|   ** Complete list of available subcommands:                  |
|                                                               |
|    RSFILTER VARIABLE IS v.                        @           |
|             NEW-SERIES ARE v1,v2.                 @           |
|             SEASONALITY is i. (PERIOD IS i.)      @           |
|             DFORDERS ARE  i1, i2, --- .           @           |
|             NODFORDERS i1, i2, --- .              @           |
|             SPAN IS i1, i2.                                   |
|                                                               |
|      Required subcommand:  VARIABLE and NEW-SERIES            |
|                                                               |
|      Legend:  v -- variable name;    i -- integer value;      |
|               w -- keyword                                    |
+---------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the name of the time series that will be used for model identification. It is a required subcommand.

NEW-SERIES subcommand

The NEW-SERIES subcommand is used to specify the names of variables to store the R (nonseasonal) series and S (seasonal) series. The S series will not be generated if the SEASONALITY (or PERIOD) subcommand is not specified. It is a required subcommand.

SEASONALITY subcommand

The SEASONALITY subcommand is used to specify the potential seasonality that the series may possess. The seasonality is 4 for quarterly data, 12 for monthly data, and so on. If a seasonality is specified but in fact the series is nonseasonal, an appropriate nonseasonal model can still be identified from the R series (assuming that the series has a reasonable number of observations). It is safe to specify the potential seasonality if the series is medium to long in length.

PERIOD subcommand

The PERIOD subcommand serves the same purpose as the SEASONALITY subcommand. Either PERIOD or SEASONALITY may be used to specify the periodicity or seasonality of a time series, but not both.

DFORDER subcommand

The DFORDER subcommand is used to specify the differencing order(s) that must be included in the final ARIMA model. In addition to the specified differencing(s), other differencing orders may be included if they are found to be necessary. By default, the differencing orders of a time series are automatically determined by the RSFILTER command. This imposed specification of differencing order(s) may be particularly useful when a time series is short.

NODFORDERS subcommand

The NODFORDERS subcommand is used to exclude specific differencing order(s) from being used in the RSFILTER command. By default, the differencing orders for a time series are automatically determined by the RSFILTER command.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which the data are used to generate the R and S series. The default is the maximum span available for the series.

TARESTIM

The TARESTIM command is used to estimate the model parameters of a threshold autoregressive (TAR) model.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|      TARESTIM  MODEL model-name.                         @    |
|                REGIME IS v1.                             @    |
|                HOLD RESIDUALS(v).                             |
|                                                                |
|   ** Complete list of available subcommands:                   |
|                                                                |
|      TARESTIM  MODEL model-name.                         @    |
|                REGIME IS v1.                             @    |
|                SPAN IS i1, i2.                           @    |
|                METHOD IS w.                              @    |
|                STOP ARE MAXIT(i), LIKELIHOOD(r1),        @    |
|                       ESTIMATE(r2), STDEV(r3).           @    |
|                OUTPUT IS LEVEL(w), PRINT(w1, w2, - - -), @    |
|                        NOPRINT(w1, w2, - - -).           @    |
|                HOLD RESIDUALS(v), FITTED(v), VARIANCE(v).     |
|                                                                |
|      Required subcommand:   MODEL                              |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;         |
|               w -- keyword;        r -- real value             |
+----------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the label (name) of the model to be estimated. The label must be one specified in a previous TSMODEL command. It is a required subcommand.

REGIME subcommand

The REGIME subcommand is used to specify the binary indicator variable that marks the observations associated with a regime.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series.

METHOD subcommand

The METHOD subcommand is used to specify the method for the computation of the likelihood function used in model estimation. The keyword may be CONDITIONAL for the "conditional" likelihood or EXACT for the "exact" likelihood function. The default is CONDITIONAL.

STOP subcommand

The STOP subcommand is used to specify the stopping criterion for the nonlinear estimation of parameters.

Estimation is terminated when the relative change in the value of the likelihood function of parameter estimates between two successive iterations is less than or equal to the convergence criterion, or if the maximum number of iterations is reached.

The argument, i, for the keyword MAXIT specifies the maximum number of iterations. The default is i=10.

The argument, r1, for the keyword LIKELIHOOD specifies the value of the relative convergence criterion on the likelihood function. The default is r1 = 0.0001.

The argument, r2, for the keyword ESTIMATE specifies the value of the relative convergence criterion on the parameter estimates. The default is r2 = 0.001.

The argument, r3, for the keyword STDEV specifies the value of the relative convergence criterion on the estimate of the standard deviation in the iteration. This criterion is employed by the SCA System if a constant term is present in the model. The default is r3=0.001 when a constant term is present and the criterion is disabled otherwise. The criterion can be disabled by the user by specifying a negative value for r3. The criterion is enabled if a positive value is specified for r3, even if no constant term is present.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed for computed statistics. Control is achieved in a two stage procedure. First a basic LEVEV of output is specified. Output may then be increased from this level by use of PRINT, or decreased from this level by use of NOPRINT.

The keywords for LEVEL and output printed are:

BRIEF:	estimates and their related statistics only
NORMAL:	RCORR
DETAILED:	RCORR, ITERATION, and CORR

where the reserved words on the right denote:

RCORR:	the reduced correlation matrix for the parameter estimates
ITERATION:	the parameter and covariance estimates for each iteration
CORR:	the correlation matrix for the parameter estimates

These reserved words are also keywords for PRINT and NOPRINT. The default for LEVEL is NORMAL, or the level specified in the PROFILE command.

Note: Reduced correlation matrix refers to the display of the correlation matrix of parameter estimates in which all values have no more than two decimal places with those estimates within two standard errors of zero displayed as dots, "." .

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the statistics below will be retained after the command is used. The values that may be retained are:

RESIDUALS:	the residual series without outlier adjustment
FITTED:	the one step-ahead forecasts (fitted values) of the series
VARIANCE:	the variance of the noise

TARFORECAST

The TARFORECAST command is used to compute the forecasts for a threshold autoregressive (TAR) model possessing one or more regimes. Note that there are two alternative syntax specifications. If the WEIGHT subcommand is used, the TARFORECAST command syntax and functionalities are identical to the WFORECAST command.

(1) SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|   TARFORECAST  MODELS ARE model-name1, model-name2, ---.  @    |
|                THRESHOLD IN v.                                 |
|                                                                |
|  ** Complete list of available subcommands:                    |
|                                                                |
|   TARFORECAST  MODELS ARE model-name1, model-name2, ---.  @    |
|                THVARIABLE IS v.                           @    |
|                THRESHOLD IN v.                            @    |
|                ORIGIN IS i.                               @    |
|                NOFS IS i.                                 @    |
|                OUTPUT IS PRINT(w1,w2,---),                @    |
|                          NOPRINT(w1,w2,---),              @    |
|                HOLD FORECASTS(v), STD_ERRS(v).                 |
|                                                                |
|     Required subcommand:  MODEL and THRESHOLD                  |
|                                                                |
|     Legend:  v -- variable name;    i -- integer value;        |
|              w -- keyword                                      |
+----------------------------------------------------------------+
```

(2) SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|   TARFORECAST  MODELS ARE model-name1, model-name2, ---.  @    |
|                WEIGHTS IN v1, v2, ---.                         |
|                                                                |
|  ** Complete list of available subcommands:                    |
|                                                                |
|   TARFORECAST  MODELS ARE model-name1, model-name2, ---.  @    |
|                THVARIABLE IS v.                           @    |
|                WEIGHTS IN v1, v2, ---.                         |
|                ORIGIN IS i.                               @    |
|                NOFS IS i.                                 @    |
|                OUTPUT IS PRINT(w1,w2,---),                @    |
|                          NOPRINT(w1,w2,---),              @    |
|                HOLD FORECASTS(v), STD_ERRS(v).                 |
|                                                                |
|     Required subcommand:  MODEL and WEIGHTS                    |
|                                                                |
|     Legend:  v -- variable name;    i -- integer value;        |
|              w -- keyword                                      |
+----------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the names (labels) of the AR models for the regimes of a TAR model. The order of the models must correspond to the regimes in ascending order as defined in the THRESHOLD subcommand. It is a required subcommand.

THVARIABLE subcommand

The THVARIABLE subcommand is used to specify the name (label) of the threshold variable. The default is the dependant variable in the regime models. The threshold variable can also be specified as any other variable that may or may not be part of the model.

THRESHOLD subcommand

The THRESHOLD subcommand is used to specify a vector that contains the threshold lag (delay) and the cut-off values (threshold values) that define the regimes of a TAR model. The first element of the vector specifies the threshold lag. The remaining elements specify the threshold values pertaining to the model regimes. More than one threshold value may be specified, and they must be specified in ascending order. The WEIGHTS subcommand must not be specified if the THRESHOLDS subcommand is used.

WEIGHTS subcommand

The WEIGHTS subcommand is used for the alternative syntax specification for the WFORECAST command. The THRESHOLD subcommand must not be specified if the WEIGHTS subcommand is used. By specifying the WEIGHTS subcommand, the TARFORECAST command is the same as the WFORECAST command. The WEIGHTS subcommand is used to specify the weight variables for the models used for forecasting. The minimum length of the weight variables must be equal to the number of forecasts specified in the NOFS subcommand. The values for the forecast weights are typically between 0 and 1, where 0 receives no weight. The weighted forecasts are computed as the proportional contribution of each forecast based on the given weights. For example, if two forecasting models are used in weighted forecasting and the weights for both models are set to 1 at a particular forecast period, the proportional contribution of forecasts for that period is 1/(1+1) or 50%. This can be more generally stated as weight(i) divided by the sum of all weights for a particular period.

ORIGIN subcommand

The ORIGIN subcommand is used to specify the time origin for forecasting. The default is the last observation of the series.

NOFS subcommand

The NOFS subcommand is used to specify the number of forecasts to be generated. The default is 24 forecasts.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed or plotted for computed statistics. Control is achieved by increasing or decreasing the basic level of output by use of PRINT or NOPRINT, respectively. The keyword for PRINT and NOPRINT is:

FORECAST:	forecast values for each time origin

The default condition is PRINT(FORECAST).

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parenthesis. Default is that none of the values of the above statistics will be retained after the command is executed. The values that may be retained are:

FORECASTS:	forecasts at the last time origin
STD_ERRS:	standard errors of the forecasts at the last time origin

TARTEST

The TARTEST command serves two purposes, (1) testing a time series for nonlinearity based on the TAR-F test and other types of tests; and (2) to determine threshold values for a TAR model based on the parameter estimates of the arranged autoregression (AAR) or reverse arranged autoregression (RAAR) method. The latter function can also be performed using the THMEXPLORE command.

If the TARTEST command is being used to determine the threshold values for a TAR model, only a single value in the THLAG subcommand should be specified. In this case, the HOLD subcommand can be used to save the AR estimates, their t-values, and residual standard errors to explore potential threshold values. If a set of THLAGS is specified, it is assumed that the user is searching for the best threshold lag (delay) based on the statistics of nonlinearity tests.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|  (1) Nonlinearity tests                                        |
|                                                                |
|      TARTEST   VARIABLE IS v.                          @      |
|                ARLAGS ARE i1,i2, --- .                 @      |
|                THLAGS ARE i1,i2, --- .                 @      |
|                METHOD IS w.                            @      |
|                STANDARDIZED./NO STANDARDIZED.          @      |
|                PSTART r.                               @      |
|                SPAN IS i1, i2.                                |
|                                                                |
|  (2) Obtaining parameter estimates for examination of          |
|      potential threhold values                                 |
|                                                                |
|      TARTEST   VARIABLE IS v.                          @      |
|                ARLAGS ARE i1,i2, --- .                 @      |
|                THLAG IS i1.                            @      |
|                METHOD IS w.                            @      |
|                STANDARDIZED./NO STANDARDIZED.          @      |
|                PSTART r.                               @      |
|                SPAN IS i1, i2.                         @      |
|                HOLD THVARIABLE(v1,v2),PHI(v),TPHI(v),  @      |
|                     SIGMAHAT(v).                               |
|                                                                |
|      Required subcommand:   VARIABLE, THLAG, and ARLAGS        |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;         |
|               w -- keyword;        r -- real value             |
+----------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the name of the series for which the nonlinearity tests will be conducted, or the threshold values will be explored. It is a required subcommand.

ARLAGS subcommand

The ARLAGS subcommand is used to specify the lag orders of the underlying AR model used for nonlinearity tests. The lags must be consecutive and start with 1 if ALLTESTS is specified in the METHOD subcommand. Otherwise the highest AR order model is used for the ALLTESTS option. Skipping lags are allowed if the AAR or RAAR method is specified. It is a required subcommand.

THLAGS subcommand

The THLAGS subcommand is used to specify the threshold lags (delays) considered for the threshold nonlinearity test. If a single threshold lag is specified, then it assumed that the TARTEST command is being used to explore potential threshold values for a TAR model. This is a required subcommand.

METHOD subcommand

The METHOD subcommand is used to specify the method for the nonlinearity test. The keywords are AAR (arranged autoregression), RAAR (reverse arranged autoregression), and ALLTESTS. If the TARTEST is used for computation of parameter estimates for determination of threshold values, then only AAR or RAAR can be specified. The default is the AAR method.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series.

STANDARDIZED subcommand

The STANDARDIZED subcommand is used to specify if the predictive residuals are standardized in the computation of the F-statistics or not. The default is STANDARDIZED.

PSTART subcommand

The PSTART subcommand is used to specify the portion of the data used for initial estimates of the model parameters. The default is 0.10 (i.e., 10% of the data).

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the above statistics will be retained after the command is used. The values that may be retained are:

THVARIABLE:	arranged threshold variable (v1), and the time index of the sorted variable.
PHI:	a matrix that holds the parameter estimates for the autoregressive parameters (in ascending lag order) and the constant term (the last column of the matrix) in the model. The parameter estimates in each column of the matrix can be easily obtained by a simple analytic statement. For example, the statement

PHI1=PHIMTX(.,1) will assign the first column of PHIMTX to the vector variable PHI1.

TPHI:	a matrix that holds the t-values for the autoregressive and constant terms of the model. Its use is similar to the keyword PHI.
SIGMAHAT:	a vector variable that holds the incremental residual standard deviations from the model estimations.

TARXTEST

The TARXTEST command is used to test the nonlinearity of a time series and obtain the best threshold lag (delay) when the threshold variable is an exogenous variable, rather than a lag of the dependent variable.

SYNTAX in terms of subcommands and associated argument type(s)

```
+-------------------------------------------------------------+
|                                                             |
|      TARXTEST  VARIABLE IS v.                          @    |
|                ARLAGS ARE i1,i2, --- .                 @    |
|                THVARIABLE IS v.                        @    |
|                THLAGS ARE i1,i2, ... .                 @    |
|                METHOD IS w.                            @    |
|                STANDARDIZED./NO STANDARDIZED.          @    |
|                PSTART r.                               @    |
|                SPAN IS i1, i2.                              |
|                                                             |
|     Required subcommand: VARIABLE, THLAG, THVARIABLE, and   |
|                          ARLAGS                             |
|                                                             |
|      Legend:  v -- variable name;  i -- integer value;      |
|               w -- keyword;        r -- real value          |
+-------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the name of the series for which the nonlinearity test will be conducted. It is a required subcommand.

ARLAGS subcommand

The ARLAGS subcommand is used to specify the lag orders of the underlying AR model used for nonlinearity tests. Skipping lags are allowed. This is a required subcommand.

THVARIABLE subcommand

The THVARIABLE subcommand is used to specify the time series served as the threshold variable. The threshold variable is an exogenous variable in this command. This is a required subcommand.

THLAGS subcommand

The THLAGS subcommand is used to specify the threshold lags (delays) considered for the threshold nonlinearity test. Lag 0 is allowed in this case. This is a required subcommand.

METHOD subcommand

The METHOD subcommand is used to specify the method for the nonlinearity test. The keywords supported are AAR (arranged autoregression) and RAAR (reverse arranged autoregression). The default is the AAR method.

STANDARDIZED

The STANDARDIZED subcommand is used to specify if the predictive residuals are standardized in the computation of the F-statistics or not. The default is STANDARDIZED.

PSTART subcommand

The PSTART subcommand is used to specify the portion of the data used for initial estimates of the model parameters. The default is 0.10 (i.e., 10% of the data).

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series.

THMTEST

The THMTEST command is used to test nonlinearity relationships between time series and obtain the best threshold lag (delay). It is assumed that the specified model has a white noise process.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|      THMTEST   VARIABLE IS v.                            @    |
|               XVARIABLES ARE v1, v2, ... .              @    |
|               THVARIABLE IS v.                          @    |
|               THLAGS ARE i1,i2, ... .                   @    |
|               METHOD IS w.                              @    |
|               STANDARDIZED./NO STANDARDIZED.            @    |
|               PSTART r.                                 @    |
|               SPAN IS i1, i2.                                 |
|                                                                |
|     Required subcommand: VARIABLE, THLAG, and THVARIABLE       |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;         |
|               w -- keyword;        r -- real value            |
+----------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the name of the dependent series for which the nonlinearity relationships will be examined. It is a required subcommand.

XVARIABLES subcommand

The XVARIABLES subcommand is used to specify the names of the exogenous series in the model for nonlinearity test. This is a required subcommand.

THVARIABLE subcommand

The THVARIABLE subcommand is used to specify the time series served as the threshold variable. The threshold variable can be the dependent or any exogenous series. This is a required subcommand.

THLAGS subcommand

The THLAGS subcommand is used to specify the threshold lags (delays) considered for the threshold nonlinearity test. Lag 0 is allowed in this case. This is a required subcommand.

METHOD subcommand

The METHOD subcommand is used to specify the method for the nonlinearity test. The keywords supported are AAR (arranged autoregression) and RAAR (reverse arranged autoregression). The default is the AAR method.

STANDARDIZED

The STANDARDIZED subcommand is used to specify if the predictive residuals are standardized in the computation of the F-statistics or not. The default is STANDARDIZED.

PSTART subcommand

The PSTART subcommand is used to specify the portion of the data used for initial estimates of the model parameters. The default is 0.10 (i.e., 10% of the data).

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series.

THMEXPLORE

The THMEXPLORE command is used to provide parameter estimates of a model according to the order of the sorted threshold variable. The threshold variable may be sorted in ascending or descending order. A moving-window regression estimation is used to explore the potential nonlinearity and threshold values of the model parameters. The THMEXPLORE command can be used with an autoregressive model or a transfer function model. For autoregressive models, this command provides AAR (arranged autoregression), RAAR (reverse arranged autoregression), and LAR (local arranged autoregression) methods for the identification of threshold values.

In this command, the model parameters to be estimated must be specified with names (labels). For example, Y=CNST+(BETA1)X1+NOISE where CNST and BETA1 are the parameter estimate names associated with the constant and input variable X1. Furthermore, the model must be expressed in linear regression form using the TSMODEL command with white noise and without any backshift operator. All lagged variables must be created prior to using this command via the LAG command.

The parameter estimates for each data window are stored in the associated names for the parameters as vectors. The lengths of the resulting vectors for the parameter estimates are the same as the length of the output variable. The t-values for the parameter estimates are also stored for each window-estimation using the root word of the parameter label preceded by an underscore character (e.g., _CNST and _BETA1). There are estimates at the beginning of the series that are missing due to initial window size. Any parameter estimates that are unavailable (missing) are padded with the missing value code.

SYNTAX in terms of subcommands and associated argument type(s)

```
+---------------------------------------------------------------+
|                                                               |
|  THMEXPLORE MODEL model-name.                          @   |
|             THVARIABLE IS v.                          @   |
|             THLAG IS i.                               @   |
|             WINDOWSIZE i1,i2.                         @   |
|             BEGIN i1,i2.                              @   |
|             DESCENDING./NO DESCENDING.                @   |
|             HOLD THVARIABLE(v),SEQUENCE(v),VARIANCE(v).   |
|                                                               |
|   ** Complete list of available subcommands:                  |
|                                                               |
|  THMEXPLORE MODEL model-name.                          @   |
|             THVARIABLE IS v.                          @   |
|             THLAG IS i.                               @   |
|             WINDOWSIZE i1,i2.                         @   |
|             BEGIN i1,i2.                              @   |
|             DESCENDING./NO DESCENDING.                @   |
|             SPAN IS i1, i2.                           @   |
|             OUTPUT IS LEVEL(w),PRINT(w1,w2,---),      @   |
|                                NOPRINT(w1,w2,---).    @   |
|             HOLD THVARIABLE(v),SEQUENCE(v),VARIANCE(v).   |
|                                                               |
|      Required subcommand:  MODEL and THLAG                    |
|                                                            |
|      Legend:  v -- variable name;    i -- integer value;    |
|              w -- keyword                                   |
+------------------------------------------------------------+
```

<u>Subcommand descriptions</u>

MODEL subcommand

The MODEL subcommand is used to specify the name (label) of the potential threshold model to be explored. The name must be one specified in a previous TSMODEL or IARIMA command. It is a required subcommand.

THVARIABLE subcommand

The THVARIABLE subcommand is used to specify the time series serving as the threshold variable. The threshold variable can be an external variable that is not explicitly specified in the model. The default is the dependent variable in the model.

THLAG subcommand

The THLAG subcommand is used to specify the threshold lag (delay) for the threshold variable. The user may obtain this value using the TARTEST or TARXTEST command. Lag 0 is allowed if the threshold variable is not the dependent variable in the model. It is a required subcommand.

WINDOWSIZE subcommand

The WINDOWSIZE subcommand is used to specify the size of the data window employed in the sorted regression estimation. If i2 is specified, it indicates the increment for the data span of the next data window in sorted regression estimation. It is required to specify the value of i1. The value of i2 is set to the default of 1 if it is not specified.

For autoregressive models, if the window size is set to the maximum data length and the threshold variable is sorted in ascending order, this command produces estimates using the AAR method. Conversely, this command produces estimates using the RAAR method if the threshold variable is in descending order. The LAR method is implied if the window size is not the maximum of the data length.

BEGIN subcommand

The BEGIN subcommand is used to specify the data index (i1) where the first moving-window estimation begins. The default sets i1 equal to the value of specified WINDOWSIZE. The code 0 can also be used to indicate that the default value for i1 is the WINDOWSIZE. The value i2 is used to indicate where the moving-window estimation ends. The default is the maximum data length. The code 0 may also be specified to set the default of i2 equal to the maximum data length.

This subcommand must be specified if the WINDOWSIZE is set to the maximum data length.

DESCENDING subcommand

The DESCENDING subcommand is used to specify if the sorted regression estimation will be performed according the ascending or descending order of the threshold variable. The default is ascending order (i.e., NO DESCENDING).

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series. The SPAN subcommand will reduce the data prior to applying the settings in the BEGIN subcommand.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output displayed for selected statistics. Control is achieved in a two stage procedure. First, a basic LEVEL of output (default NORMAL) is designated. Output may then be increased (decreased) from this level by use of PRINT (NOPRINT).

The keywords for LEVEL and output displayed are:

BRIEF:	display information for the estimates and their t-statistics
NORMAL:	same as BRIEF
DETAILED:	same as BRIEF

where the keywords on the right denote:

CORR:	the correlation matrix for the parameter estimates. It also activates the display of detailed estimation information.

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the above statistics will be retained after the command is used. The values that may be retained are:

THVARIABLE:	the sorted threshold variable
SEQUENCE:	the time index (sequence) of the sorted threshold variable
VARIANCE:	the estimated variance in each data window

TSEARCH

The TSEARCH command is used to search for the power value (lambda) in a time series power transformation given a specified univariate time series model.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|     TSEARCH  MODEL model-name.                         @       |
|              POWERS r1, r2, r3.                        @       |
|              FORM v1, v2.                              @       |
|              SPAN i1, i2, i3.                          @       |
|              WITHIN-RMSE v1, v2, v3, v4.               @       |
|              POST-RMSE v1, v2, v3.                             |
|                                                                |
|                                                                |
|   ** Complete list of available subcommands:                   |
|                                                                |
|     TSEARCH  MODEL model-name.                         @       |
|              POWERS r1, r2, r3.                        @       |
|              FORM v1, v2.                              @       |
|              RETRANSFORMATION-METHOD w.                @       |
|              WEIGHTS IN v.                             @       |
|              RMSE-WEIGHTS IN v.                        @       |
|              SPAN i1, i2, i3.                          @       |
|              WITHIN-RMSE v1, v2, v3, v3.               @       |
|              POST-RMSE v1, v2, v3.                     @       |
|              METHOD IS w.                              @       |
|              STOP ARE MAXIT(i), LIKELIHOOD(r1),        @       |
|                       ESTIMATE(r2), STDEV(r3).         @       |
|              OUTPUT IS LEVEL(w), PRINT(w1, w2, - - -), @       |
|                        NOPRINT(w1, w2, - - -).         @       |
|              HOLD RESIDUALS(v), FITTED(v), VARIANCE(v).        |
|                                                                |
|      Required subcommand:   MODEL                              |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;         |
|               w -- keyword;        r -- real value             |
+----------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the label (name) of the model to be estimated. The label must be one specified in a previous TSMODEL command. It is a required subcommand.

POWERS subcommand

The POWERS subcommand is used to specify the range of power (lambda) values to include in the power transformation analysis. The value r1 is the increment of the power value, r2 the beginning value, and r3 the ending value. The default values are 0.5, -1.0 and 1.0 respectively. The number of arguments in this subcommand may be 1, 2 or 3.

FORM subcommand

The FORM subcommand is used to specify the type of transformation to be applied and whether the geometric mean is to be used to scale the data. The argument, v1, must be a pre-defined value, and v2 should be a new variable (if an existing variable is used, its value will be over-written).

The argument, v1, must pass the value 1 or 2 to control the type of transformation to be applied. The default type is Type 1. An explanation of transformation Type 1 and Type 2 is provided in the body of the text.

The second argument, v2, if specified, indicates the power transformation analysis is to be based on data scaled by its geometric mean. After the command is executed, the v2 argument will store the computed geometric mean of the series. The geometric mean is computed based on all available data in the within-sample span. If the second argument, v2, is not specified then the transformation analysis will be conducted on the original series without scaling.

RETRANSFORM-METHOD subcommand

The RETRANSFORM-METHOD subcommand is used to specify the method for computing the one-step-ahead forecasts. The method may be specified as STRAIGHT or UNBIASED. The default is STRAIGHT.

WEIGHTS subcommand

The WEIGHT subcommand is used to specify a variable containing the weight for each observation in the time series to be used in estimating the model. The weights variable must be a vector of length greater than or equal to the number of observations in the time series. A weight typically is specified in the range of 0 to 1, where 0 indicates that an observation is to be completely discounted during estimation, and 1 indicates that the observation is not discounted during estimation.

RMSE-WEIGHTS subcommand

The RMSE-WEIGHTS subcommand is used to specify a variable containing the weight for each observation in the time series to be used in computing the within-sample RMSE statistics based on the one-step-ahead forecasts. The RMSE-WEIGHTS variable must be a vector of length greater than or equal to the number of observations in the time series. A weight typically is specified in the range of 0 to 1, where 0 indicates that an observation is to be completely discounted when computing the RMSE statistic, and 1 indicates that the observation receives no discounting when computing the RMSE statistic.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices for model estimation and to compute the within-sample RMSE and post-sample RMSE of the one-step-ahead forecasts.

The arguments, i1 and i2, specify the time indices to be used for model estimation and computing the within-sample RMSE. The argument, i3, specifies the ending period used to compute the post-sample RMSE. The beginning period used to compute the post-sample RMSE is implicitly set as the value of i2+1.

WITHIN-RMSE subcommand

The WITHIN-RMSE subcommand is used to specify the variables to hold information regarding the within-sample RMSE statistics computed over the range of power (lambda) values being analyzed.

The argument, v1, specifies the variable to hold the power (lambda) values searched in the power transformation analysis.

The argument, v2, specifies the variable to hold the RMSE statistics of the one-step-ahead forecasts which are re-transformed into the original scale of the time series. If the RMSE-WEIGHTS subcommand is specified, these RMSE statistics are based on the weights provided. It is recommended that the power value (lambda) be determined based on the within-sample RMSE statistics.

The argument, v3, specifies the variable to hold the RMSE statistics of the one-step-ahead forecasts based on the transformed time series. If the RMSE-WEIGHTS subcommand is specified, these RMSE statistics are based on the weights provided.

The argument, v4, specifies the variable to hold the RMSE statistics of the one-step-ahead forecasts based on the transformed time series. The RMSE statistics are based on all available data associated with the within-sample period where all observations are included (i.e., the RMSE-WEIGHT variable is ignored). If the RMSE-WEIGHTS subcommand is not specified, the RMSE statistics saved in v4 will be the same as the RMSE statistics saved in v3.

POST-RMSE subcommand

The POST-RMSE subcommand is used to specify the variables to hold information regarding the post-sample RMSE statistics computed over the range of power (lambda) values being analyzed.

The argument, v1, specifies the variable to hold the power (lambda) values searched in the power transformation analysis.

The argument, v2, specifies the variable to hold the RMSE statistics of the one-step-ahead forecasts which are re-transformed into the original scale of the time series. If the RMSE-WEIGHTS subcommand is specified, these RMSE statistics are based on the weights provided.

The argument, v3, specifies the variable to hold the RMSE statistics of the one-step-ahead forecasts based on the transformed time series. If the RMSE-WEIGHTS subcommand is specified, these RMSE statistics are based on the weights provided.

METHOD subcommand

The METHOD subcommand is used to specify the method for the computation of the likelihood function used in model estimation. The keyword may be CONDITIONAL for the “conditional” likelihood or EXACT for the “exact” likelihood function. The default is CONDITIONAL.

STOP subcommand

The STOP subcommand is used to specify the stopping criterion for the nonlinear estimation of parameters. This estimation is conditional on the most recent outlier adjustment.

Estimation is terminated when the relative change in the value of the likelihood function of parameter estimates between two successive iterations is less than or equal to the convergence criterion, or if the maximum number of iterations is reached.

The argument, i, for the keyword MAXIT specifies the maximum number of iterations. The default is i=10.

The argument, r1, for the keyword LIKELIHOOD specifies the value of the relative convergence criterion on the likelihood function. The default is r1 = 0.0001.

The argument, r2, for the keyword ESTIMATE specifies this value of the relative convergence criterion on the parameter estimates. The default is r2 = 0.001.

The argument, r3, for the keyword STDEV specifies the value of the relative convergence criterion on the estimate of the standard deviation in the iteration. This criterion is employed by the SCA System if a constant term is present in the model. The default is r3=0.001 when a constant term is present and the criterion is disabled otherwise. The criterion can be disabled by the user by specifying a negative value for r3. The criterion is enabled if a positive value is specified for r3, even if no constant term is present.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed for computed statistics. Control is achieved in a two stage procedure. First a basic LEVEL of output is specified. Output may then be increased from this level by use of PRINT, or decreased from this level by use of NOPRINT.

The keywords for LEVEL and output printed are:

BRIEF:	SUMMARY
NORMAL:	SUMMARY and MODEL
DETAILED:	SUMMARY, MODEL and FORECAST

where the reserved words on the right denote:

SUMMARY:	the lambda values and their associate RMSE's and statistics
MODEL:	estimated model under each lambda
FORECAST:	post-sample forecasts

These reserved words are also keywords for PRINT and NOPRINT. The default for LEVEL is NORMAL, or the level specified in the PROFILE command.

Note: Reduced correlation matrix refers to the display of the correlation matrix of parameter estimates in which all values have no more than two decimal places with those estimates within two standard errors of zero displayed as dots, "." .

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the statistics below will be retained after the command is used. The values that may be retained are:

RESIDUALS:	the residual series without outlier adjustment
FITTED:	the one step-ahead forecasts (fitted values) of the series
VARIANCE:	the variance of the noise
VPOWER:	the lambda values
IPOWER:	the index position of the smallest lambda value in VPOWER

TVPEXPLORE

The TVPEXPLORE command is used to provide information for time series models that may possess time-varying parameters. A moving-window estimation method is used to explore the behavior of such models over time. The TVPEXPLORE command can be used with an ARIMA or a transfer function model. In this command, the model parameters to be estimated must be specified with names (labels). For example,

Y=CNST+(0;BETA1)X1+1/(1;PHI)NOISE

where CNST, BETA1 and PHI are the names associated with the model parameters.

The parameter estimates for each data window used are stored in the associated names for the parameters as vectors. The lengths of the resulting vectors for the parameter estimates are the same as the length of the output variable. The t-values for the parameter estimates are also stored for each window estimation using the root word of the parameter label preceded by an underscore character (e.g., _CNST, _BETA1 and _PHI). There are estimates at the beginning of the series that are missing due to initial window size. Any parameter estimates that are unavailable (missing) are padded with the SCA missing value code.

SYNTAX in terms of subcommands and associated argument type(s)

```
+-------------------------------------------------------------+
|                                                             |
|  TVPEXPLORE MODEL model-name.                          @   |
|             WINDOWSIZE i1,i2.                          @   |
|             BEGIN i1,i2.                               @   |
|             HOLD VARIANCE(v).                              |
|                                                             |
|   ** Complete list of available subcommands:               |
|                                                             |
|  TVPEXPLORE MODEL model-name.                          @   |
|             WINDOWSIZE i1,i2.                          @   |
|             BEGIN i1,i2.                               @   |
|             SPAN IS i1, i2.                            @   |
|             INITIALIZED / NO INITALIZED.               @   |
|             METHOD IS w.                               @   |
|             STOP ARE MAXIT(i),LIKELIHOOD(r1),ESTIMATE(r2).@   |
|             OUTPUT IS LEVEL(w),PRINT(w1,w2,---),       @   |
|                               NOPRINT(w1,w2,---).      @   |
|             HOLD VARIANCE(v).                              |
|                                                             |
|       Required subcommand:  MODEL                          |
|                                                             |
|       Legend:  v -- variable name;    i -- integer value;  |
|                w -- keyword                                |
+-------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the name (label) of the model to be explored. The name must be one specified in a previous TSMODEL or IARIMA command. It is a required subcommand.

WINDOWSIZE subcommand

The WINDOWSIZE subcommand is used to specify the size of data window (i1) in each estimation. If i2 is specified, it indicates the increment for the time span of the next data window in time-varying parameter estimation. It is required to specify the value of i1. The value of i2 is set to the default of 1 if it is not specified.

BEGIN subcommand

The BEGIN subcommand is used to specify the time index (i1) where the first moving-window estimation begins. The default sets i1 equal to the value of specified WINDOWSIZE. The code 0 can also be used to indicate that the default value for i1 is the WINDOWSIZE. The value i2 is used to indicate where the moving-window estimation ends. The default is the maximum ending time period. The code 0 may also be specified to set the default of i2 equal to the maximum ending time period.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series. The SPAN subcommand will reduce the data prior to applying the settings in the BEGIN subcommand.

INITIALIZED subcommand

The INITIALIZED subcommand is used to indicate that each parameter in the model will be initialized (i.e., all set to 0.1 except for the constant term) or not initialized (i.e., parameter values from previous estimation used as starting values for next estimation) for the nonlinear estimation in each data window. The default is INITIALIZED.

METHOD subcommand

The METHOD subcommand is used to specify the method for the computation of the likelihood function used in model estimation. The keyword may be CONDITIONAL for the conditional" likelihood or EXACT for the "exact" likelihood function. The default is CONDITIONAL.

STOP subcommand

The STOP subcommand is used to specify the stopping criterion for the nonlinear estimation of parameters. Estimation is terminated when the relative change in the value of the likelihood function or parameter estimates between two successive iterations is less than or equal to the convergence criterion, or if the maximum number of iterations is reached.

The argument, i, for the keyword MAXIT specifies the maximum number of iterations. The default is i=10.

The argument, r1, for the keyword LIKELIHOOD specifies the value of the relative convergence criterion on the likelihood function. The default is r1 = 0.0001.

The argument, r2, for the keyword ESTIMATE specifies the value of the relative convergence criterion on the parameter estimates. The default is r2 = 0.001.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output displayed for selected statistics. Control is achieved in a two stage procedure. First, a basic LEVEL of output (default NORMAL) is designated. Output may then be increased (decreased) from this level by use of PRINT (NOPRINT).

The keywords for LEVEL and output displayed are:

BRIEF:	display information for the estimates and their t-statistics
NORMAL:	same as BRIEF
DETAILED:	ALLESTIM

where the keywords on the right denote:

ALLESTIM:	the parameter and covariance estimates for each iteration
CORR:	the correlation matrix for the parameter estimates. It also activates the display of detailed estimation information.

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the above statistics will be retained after the command is used. The values that may be retained are:

VARIANCE:	the estimated variance in each data window

UROOT

The UROOT command is used to test a time series for the presence of a unit root based on the methods of Dickey-Fuller or Phillips-Perron.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|      UROOT VARIABLE IS v1.                               @    |
|            ORDERS ARE i1,i2,---.                         @    |
|            METHOD IS w.                                       |
|                                                                |
|                                                                |
|   ** Complete list of available subcommands:                  |
|                                                                |
|      UROOT VARIABLE IS v1.                               @    |
|            ORDERS ARE i1, i2,---.                        @    |
|            METHOD IS w.                                  @    |
|            SIGNIFICANCE-LEVEL IS v1.                     @    |
|            SPAN IS i1,i2.                                     |
|                                                                |
|      Required subcommand:   VARIABLE, METHOD                  |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;        |
|               w -- keyword;        r -- real value            |
+----------------------------------------------------------------+
```

Subcommand descriptions

VARIABLE subcommand

The VARIABLE subcommand is used to specify the series for which a unit root test is conducted.

ORDERS subcommand

The ORDERS subcommand is used to specify the order(s) to be tested. The default is 0 (no additional lags) for the simple Dickey-Fuller and the simple Phillips-Perron tests, and 1 for the augmented versions of the unit root tests.

METHOD subcommand

The METHOD subcommand is used to specify the unit root test to be employed. The valid keywords are

DF:	Dickey-Fuller test
DFC:	Augmented Dickey-Fuller test with constant
DFT:	Augmented Dickey-Fuller test with constant and trend
PP:	Phillips-Perron test
PPC:	Augmented Phillips-Perron test with constant
PPT:	Augmented Phillips-Perron test with constant and trend

SIGNIFICANCE-LEVEL subcommand

The SIGNIFICANCE-LEVEL subcommand is used to specify the significance level for the unit root test. The default is set to the 0.05 level. The critical-values for the various tests are interpolated from the tables derived by Dickey and Fuller.

SPAN subcommand

The SPAN subcommand is used to specify the span of the series to be considered. The default is all available data.

WESTIM

The WESTIM command is used to perform weighted estimation for an ARIMA or transfer function model. The WESTIM command performs model estimation using conditional or exact maximum likelihood estimation. It can also perform joint model estimation with outlier adjustment if specified.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|      WESTIM  MODEL model-name.                         @      |
|              WEIGHTS IN v.                             @      |
|              SPAN IS i1, i2.                           @      |
|              HOLD RESIDUALS(v).                               |
|                                                                |
|   ** Complete list of available subcommands:                  |
|                                                                |
|      WESTIM  MODEL model-name.                         @      |
|              WEIGHTS IN v.                             @      |
|              SPAN IS i1, i2.                           @      |
|              METHOD IS w.                              @      |
|              STOP ARE MAXIT(i), LIKELIHOOD(r1),        @      |
|                       ESTIMATE(r2), STDEV(r3).         @      |
|              TYPES ARE w1, w2, - - - .                 @      |
|              DELTA IS r.                               @      |
|              OSTOP ARE MXOUTLIERS(i1), CRITICAL(r),    @      |
|                        MXESTIM(i2).                    @      |
|              NEW-SERIES IN v1, v2, v3, v4, v5.         @      |
|              OADJUSTMENT IS w.                         @      |
|              STDEV IS w(r).                            @      |
|              OUTPUT IS LEVEL(w), PRINT(w1, w2, - - -),  @      |
|                        NOPRINT(w1, w2, - - -).         @      |
|              HOLD RESIDUALS(v), FITTED(v), VARIANCE(v).       |
|                                                                |
|      Required subcommand:   MODEL, WEIGHTS                    |
|                                                                |
|      Legend:  v -- variable name;  i -- integer value;         |
|               w -- keyword;        r -- real value            |
+----------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the label (name) of the model to be estimated. The label must be one specified in a previous TSMODEL command. It is a required subcommand.

WEIGHTS subcommand

The WEIGHT subcommand is used to specify a variable containing the weight for each observation in the time series to be used in estimating the univariate time series model. The weights variable must be a vector of length greater than or equal to the number of observations in the time series. A weight typically is specified in the range of 0 to 1, where 0 indicates that an observation is to be completely discounted during estimation, and 1 indicates that the observation is not discounted during estimation.

SPAN subcommand

The SPAN subcommand is used to specify the span of time indices, i1 to i2, for which data are analyzed. The default is the maximum span available for the series.

METHOD subcommand

The METHOD subcommand is used to specify the method for the computation of the likelihood function used in model estimation. The keyword may be CONDITIONAL for the “conditional” likelihood or EXACT for the “exact” likelihood function. The default is CONDITIONAL.

STOP subcommand

The STOP subcommand is used to specify the stopping criterion for the nonlinear estimation of parameters. Estimation is terminated when the relative change in the value of the likelihood function of parameter estimates between two successive iterations is less than or equal to the convergence criterion, or if the maximum number of iterations is reached.

The argument, i, for the keyword MAXIT specifies the maximum number of iterations. The default is i=10.

The argument, r1, for the keyword LIKELIHOOD specifies the value of the relative convergence criterion on the likelihood function. The default is r1 = 0.0001.

The argument, r2, for the keyword ESTIMATE specifies this value of the relative convergence criterion on the parameter estimates. The default is r2 = 0.001.

The argument, r3, for the keyword STDEV specifies the value of the relative convergence criterion on the estimate of the standard deviation in the iteration. This criterion is employed by the SCA System if a constant term is present in the model. The default is r3=0.001 when a constant term is present and the criterion is disabled otherwise. The criterion can be disabled by the user by specifying a negative value for r3. The criterion is enabled if a positive value is specified for r3, even if no constant term is present.

TYPES subcommand

The TYPES subcommand is used to specify the types of outliers to be detected. The valid keywords are IO (innovative outlier), AO (additive outlier), LS (level shift), and TC (temporary change). The default is IO, AO, TC, and LS.

DELTA subcommand

The DELTA subcommand is used to specify the delta value employed for the TC outlier. The default is delta=0.7.

OSTOP subcommand

The OSTOP subcommand is used to specify the stopping criterion for outlier detection. Parameter estimation and outlier detection and adjustment are done iteratively. If any outlier is detected after a parameter estimation, the time series is adjusted for outliers and parameters are re-estimated. The iteration stops if the maximum number of outliers that may be adjusted is reached, if the maximum number of re-estimation of parameters is reached; or if all outlier statistics are smaller than a specified critical value.

The argument for the keyword MXOUTLIERS (i1) specifies the maximum number of outliers permitted to be detected and adjusted. The default for i1 is equal to 10% of the number of observations.

The argument for the keyword CRITICAL (r) specifies a critical value for testing the presence of outliers. It is recommended r = 3.50 for low sensitivity, r = 3.00 for medium sensitivity, and r = 2.70 for high sensitivity. The default for r is 3.0.

The argument for the keyword MXESTIM (i2) specifies the maximum number of re-estimation within each estimation of model parameter. The default for i2 is 3.

NEW-SERIES subcommand

The NEW-SERIES subcommand is used to specify the labels (names) of variables to be created for saving information during the outlier detection process. Only those results desired to be retained need be named. The default is that no variables are retained after the command is executed. The variables that may be retained (and the position a label must occupy in the subcommand) are:

v1: the name used to store residuals after the outlier adjustment

v2: the name used to store the adjusted series (i.e., the resultant series after removing the outlier effects from the original observations)

v3: the name used to store an indicator variable associated with the types of outliers, if any, found during the outlier detection process. The value of the t-th observation of this variable is 0 if the t-th value of the time series is not an outlier; 2 if it is an innovative outlier; 3 if it is an additive outlier; 4 if it is a temporary change; 5 if it is a level shift, and 1 if its valu3 is missing.

v4: the name used to store the estimates of any detected outliers

v5: the name used to store the effects of detected outliers on residuals

OADJUSTMENT subcommand

The OADJUSTMENT subcommand is used to specify the method of outlier estimation and adjustment. The keyword may be SEQUENTIAL for the sequential method, JOINT for the joint method, and NONE for no outlier detection and adjustment. The default is NONE.

STDEV subcommand

The STDEV subcommand is used to specify a method for the estimation of the standard error. TRIM(r) specifies that an r*100% trimmed standard deviation is used (i.e., the top r*100% largest observations, according to absolute values, are excluded from the computation). A specification of TRIM(0.0) indicates that the standard error is computed at each observation (residual) using all data except the current observation. TRIM(0.0) is the default. MAD(r) specifies that the median absolute deviation is used for sigma (i.e., sigma = 1.483*median absolute deviation).

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed for computed statistics. Control is achieved in a two stage procedure. First a basic LEVEL of output is specified. Output may then be increased from this level by use of PRINT, or decreased from this level by use of NOPRINT.

The keywords for LEVEL and output printed are:

BRIEF:	estimates and their related statistics only
NORMAL:	RCORR
DETAILED:	RCORR, ITERATION, and CORR

where the reserved words on the right denote:

RCORR:	the reduced correlation matrix for the parameter estimates
ITERATION:	the parameter and covariance estimates for each iteration
CORR:	the correlation matrix for the parameter estimates

These reserved words are also keywords for PRINT and NOPRINT. The default for LEVEL is NORMAL, or the level specified in the PROFILE command.

Note: Reduced correlation matrix refers to the display of the correlation matrix of parameter estimates in which all values have no more than two decimal places with those estimates within two standard errors of zero displayed as dots, "." .

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parentheses. The default is that none of the values of the statistics below will be retained after the command is used. The values that may be retained are:

RESIDUALS:	the residual series without outlier adjustment
FITTED:	the one step-ahead forecasts (fitted values) of the series
VARIANCE:	the variance of the noise

WFORECAST

The WFORECAST command is used to compute weighted forecasts based on the user specified models. The specified models may include both ARIMA and transfer function models. This command can be used in conjunction with the WESTIM, ESTIM, or other single-equation estimation command in the SCA Statistical System.

SYNTAX in terms of subcommands and associated argument type(s)

```
+----------------------------------------------------------------+
|                                                                |
|     WFORECAST  MODELS ARE model-name1, model-name2, ---.  @    |
|                WEIGHTS ARE v1, v2, ---.                        |
|                                                                |
|  ** Complete list of available subcommands:                    |
|                                                                |
|     WFORECAST  MODELS ARE model-name1, model-name2, ---.  @    |
|                WEIGHTS ARE v1, v2, ---.                   @    |
|                ORIGIN IS i.                               @    |
|                NOFS IS i.                                 @    |
|                IARIMA ARE v1(model-name),                 @    |
|                           v2(model-name), --- .           @    |
|                OUTPUT IS PRINT(w1,w2,---),                @    |
|                          NOPRINT(w1,w2,---),              @    |
|                HOLD FORECASTS(v), STD_ERRS(v).                 |
|                                                                |
|     Required subcommand:  MODEL and WEIGHTS                    |
|                                                                |
|     Legend:  v -- variable name;    i -- integer value;        |
|              w -- keyword                                      |
+----------------------------------------------------------------+
```

Subcommand descriptions

MODEL subcommand

The MODEL subcommand is used to specify the names (labels) of the ARIMA or transfer function models to be used for forecasting. The models must be specified in a previous TSMODEL command.

WEIGHTS subcommand

The WEIGHTS subcommand is used to specify the weight variables for the models used for forecasting. The minimum length of the weight variables must be equal to the number of forecasts specified in the NOFS subcommand. The values for the forecast weights are typically between 0 and 1, where 0 receives no weight. The weighted forecasts are computed as the proportional contribution of each forecast based on the given weights. For example, if two forecasting models are used in weighted forecasting and the weights for both models are set to 1 at a particular forecast period, the proportional contribution of forecasts for that period is 1/(1+1) or 50%. This can be more generally stated as weight(i) divided by the sum of all weights for a particular period.

ORIGIN subcommand

The ORIGIN subcommand is used to specify the time origin for forecasting. The default is the last observation of the series.

NOFS subcommand

The NOFS subcommand is used to specify the number of forecasts to be generated. The default is 24 forecasts.

IARIMA subcommand

The IARIMA subcommand is used to specify the label associated with the ARIMA model of each stochastic input series. The variable name of each stochastic input series must be entered and in parentheses the name (label) for its ARIMA model.

OUTPUT subcommand

The OUTPUT subcommand is used to control the amount of output printed or plotted for computed statistics. Control is achieved by increasing or decreasing the basic level of output by use of PRINT or NOPRINT, respectively. The keyword for PRINT and NOPRINT is:

FORECAST: forecast values for each time origin

The default condition is PRINT(FORECAST).

HOLD subcommand

The HOLD subcommand is used to specify those values computed for particular functions to be retained in the workspace until the end of the session. Only those statistics desired to be retained need be named. Values are placed in the variable named in parenthesis. Default is that none of the values of the above statistics will be retained after the command is executed. The values that may be retained are:

FORECASTS:	forecasts at the last time origin
STD_ERRS:	standard errors of the forecasts at the last time origin

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